18.03SC Practice Problems 11

Exponential and sinusoidal input signals

1. Find A so that $A\sin(3t)$ is a solution of $\ddot{x} + 4x = \sin(3t)$.

What is the general solution?

2. For $\omega \geq 0$, find A such that $A\cos(\omega t)$ is a solution of $\ddot{x} + 4x = \cos(\omega t)$.

Graph the input signal $\cos(\omega t)$ and the solution $A\cos(\omega t)$ for $\omega = 0$, $\omega = 1$, and $\omega = 3$.

Sketch a graph of A as a function of ω, as ω ranges from 0 to 5. Where does resonance occur? What is the significance of the sign of A?

3. Find an exponential solution of $\dfrac{d^4x}{dt^4} - x = e^{-2t}$.

4. Find a sinusoidal solution of $\dfrac{d^4x}{dt^4} - x = \cos(2t)$.

5. Find the general solution of the differential equations in **(3)** and **(4)**.

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