18.03SC Unit 2 Exam

1. (a) For what value of k is the system represented by $\ddot{x} + \dot{x} + kx = 0$ critically damped? [8]

(b) For k greater than that value, is the system overdamped or underdamped? [4]

(c) Suppose a solution of $\ddot{x} + \dot{x} + kx = 0$ vanishes at $t = 1$, and then again for $t = 2$ (but not in between). What is k? [8]

2. (a) Find a solution of $\ddot{x} + x = 5te^{2t}$. [10]

(b) Suppose that $y(t)$ is a solution of the same equation, $\ddot{x} + x = 5te^{2t}$, such that $y(0) = 1$ and $\dot{y}(0) = 2$. (This is probably *not* the solution you found in **(a)**.) Use $y(t)$ and other functions to write down a solution $x(t)$ such that $x(0) = 3$ and $\dot{x}(0) = 5$. [10]

3. (a) Consider the equation $\ddot{x} + b\dot{x} + kx = \cos(\omega t)$. We will vary the spring constant but keep b fixed. For what value of k is the amplitude of the sinusoidal solution of $\ddot{x} + b\dot{x} + kx = \cos(\omega t)$ maximal? (Your answer will be a function of ω and may depend upon b as well.) [10]

(b) (Unrelated to the above.) Find the general solution of $\dfrac{d^3x}{dt^3} - \dfrac{dx}{dt} = 0$. [10]

4. A certain system has input signal y and system response x related by the differential equation $\ddot{x} + \dot{x} + 6x = 6y$. It is subjected to a sinusoidal input signal.

(a) Calculate the complex gain $H(\omega)$. [10]

(b) Compute the gain at $\omega = 2$. [5]

(c) Compute the phase lag at $\omega = 2$. [5]

5. Suppose that $\frac{1}{2}t\sin(2t)$ is a solution to a certain equation $m\ddot{x} + b\dot{x} + kx = 4\cos(2t)$.

(a) Write down a solution to $m\ddot{x} + b\dot{x} + kx = 4\cos(2t-1)$. [4]

(b) Write down a solution to $m\ddot{x} + b\dot{x} + kx = 8\cos(2t)$. [4]

(c) Determine m, b, and k. [12]