Applet Exploration: Trigonometric Identity

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Start by opening the *Trigonometric Identity* applet from the *Mathlets Gallery*.

This mathlet illustrates sinusoidal functions and the trigonometric identity

$$a\cos(\omega t) + b\sin(\omega t) = A\cos(\omega t - \phi), \quad \text{where } a + ib = Ae^{i\phi}.$$

That is, (A, ϕ) are the polar coordinates of (a, b).

The sinusoidal function $A\cos(\omega t - \phi)$ is drawn here in red. A and ϕ are the amplitude and phase lag of the sinusoid. They are both controlled by sliders.

1. The phase lag ϕ measures how many radians the sinuoid falls behind the standard sinusoid, which we take to be the cosine. So when $\phi = \pi/2$ you have the sine function. Verify this in the applet.

2. The final parameter is ω, the angular frequency. High frequency means the waves come faster. Frequency zero means constant. Play with the ω slider and understand this statement. Return the angular frequency to 2.

3. The trigonometric identity shows the remarkable fact that the sum of any two sinuoidal functions of the same frequency is again a sinusoid of the same frequency.

Use the a and b sliders to select coefficients for $\cos(\omega t)$ and $\sin(\omega t)$. The a slider modifies the yellow cosine curve in the window at bottom and the b slider modifies the blue sine curve. Notice that the sum of $a\cos(t)$ and $b\sin(t)$ is displayed in the top window in green (which is a combination of blue and yellow). There it is! - the linear combination is again sinusoidal, or at least appears to be.

4. The window at the right shows the two complex numbers $a + ib$ and $Ae^{i\phi}$. The sinusoidal identity says that the green and red sinusoids will coincide exactly when the complex numbers $a + ib$ and $Ae^{i\phi}$ coincide. Verify this on the applet by pickong values of A and ϕ. and then adjusting a and b until the green and red sinusoids are the same.