First Order Autonomous DEs': Introduction

In the last topic of this unit we will study **autonomous first order differential equations**. These are (in general) nonlinear equations of the form

$$\dot{x} = f(x).$$

(Compare this with the general first order ODE $\dot{x} = f(x, t)$.)   The word autonomous means self governing and indicates that the rate of change of x is governed by x itself and is not dependent on time.

Autonomous systems have the following properties:

1. They model conditions which are constant in time, though they may depend on the current value of x.

2. They are separable.

3. They can be hard to integrate.

4. We can say a lot about them without solving them.

5. They are time invariant: if $y(t)$ is a solution then so is $y(t - t_0)$ for any value of t_0.

Our goal will be to learn to get qualitative information about the solutions without actually finding them. For example, in population models we might want to know if the population stabilizes, crashes or explodes. For physical models, we may want to know if the system is self correcting or if it can, all by itself, fall apart.

To visualize the qualitative results we will use the *phase line. This is a* very simple one dimensional plot consisting of critical points and arrows.

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