Exponential Input; Gain and Phase Lag: Introduction

The case of sinusoidal input is of great importance in applications. A sinusoidal function is a pure oscillation like $\cos(\omega t)$ or $\sin(\omega t)$, or more generally, $A\cos(\omega t - \phi)$. (As you can see, the last form includes both of the previous two by letting $A = 1$ and $\phi = 0$ or $\pi/2$).

In the temperature model, sinusoidal input could represent the diurnal (day and night) varying of outside temperature. In the concentration model it could represent the diurnal varying of the level of some hormone in the bloodstream, or the varying concentration in a sewage line of some waste product produced periodically by a manufacturing process.

In this session we are going to restrict our attention to first order constant coefficient ODE's. Before looking at sinusoidal input we will look at exponential input. For constant coefficient equations we will be able to use the *method of optimism* to find a solution without having to compute integrals. For sinusoidal input, we will use Euler's formula to convert sinusoids to *complex* exponentials, and so our solutions for exponential input will apply to sinusoids as well.

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