18.034 Honors Differential Equations
Spring 2009

18.034 Midterm #3 Name:

1. (a) (15 points) If $f' \in E$ and f is continuous, show that $\lim_{s\to\infty} sF(s) = f(0)$.

(b) (5 points) Can $F(s) = 1$ be the Laplace transform of a function $f \in E$?

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2. (a) (10 points) Show that the solution of the initial value problem

$$y'' + 2y' + 2y = f(t), \qquad y(0) = y'(0) = 0$$

is

$$y(t) = \int_0^t e^{-(t-t_1)} f(t_1) \sin(t - t_1) dt_1.$$

(b) (10 points) Show that if $f(t) = \delta(t - \pi)$ then the solution of the initial value problem in part (a) is $y(t) = h(t - \pi)e^{-(t-\pi)} \sin(t - \pi)$.

3. Consider two vectors $\vec{y}_1(t) = (t^2, 2t)$ and $\vec{y}_2(t) = (e^t, e^t)$.
(a) (10 points) In which intervals are $\vec{y}_1$ and $\vec{y}_2$ linearly independent?

(b) (10 points) Find a system of differential equations satisfied by $\vec{y}_1$ and $\vec{y}_2$.

4. Let $A = \begin{pmatrix} -1 & -1 \\ 2 & -1 \end{pmatrix}$.

(a) (10 points) Find eigenvalues and eigenvectors of A.

(b) (10 points) Find the general solution of

$$\begin{pmatrix} x \\ y \end{pmatrix}' = A \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t}.$$

5. Let $A = \begin{pmatrix} 1 & -4 \\ 4 & -7 \end{pmatrix}$.

(a) (8 points) Find eigenvalues and eigenvectors of A.

(b) (7 points) Find the solution of the initial value problem

$$\begin{pmatrix} x \\ y \end{pmatrix}' = A \begin{pmatrix} x \\ y \end{pmatrix}, \qquad \begin{pmatrix} x(0) \\ y(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}.$$

(continued to the next page)

(c) (5 points) Describe how the solution behaves as $t \to \infty$.

(d) (extra credits) Compute e^{At}.