18.034, Honors Differential Equations
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Recitation Suggestions
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Next time I will prove the generalized eigenspaces give a direct sum decomposition. Then I will explain Jordan normal form and how this gives the complete solution of a linear system of 1^{st} order ODE's over $\mathbb{C}$ (and then explain what needs to done $/\mathbb{R}$).

Until then, please keep doing examples of finding generalized eigenvectors and using the usual eigenvectors in simple cases (i.e. when generalized eigenspaces = eigenspaces) to solve systems of 1^{st} order ODE's.

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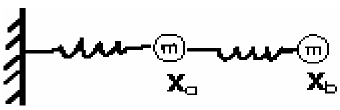
Examples: (1) $y' = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} y.$ $\qquad p_A(\lambda) = (\lambda - a)^2 + b^2,$ $\qquad \lambda = a \pm bi$

$\lambda = a + bi \ : \quad v = \begin{pmatrix} 1 \\ i \end{pmatrix}; \qquad \lambda = a - bi \ : \quad v = \begin{pmatrix} 1 \\ -i \end{pmatrix}.$

$$y = A\begin{pmatrix} 1 \\ i \end{pmatrix} e^{at} e^{ibt} + B\begin{pmatrix} 1 \\ -i \end{pmatrix} e^{at} e^{-ibt}$$

$$\rightsquigarrow \quad y = C_1 \begin{pmatrix} \cos(bt) \\ -\sin(bt) \end{pmatrix} e^{at} + C_2 \begin{pmatrix} \sin(bt) \\ \cos(bt) \end{pmatrix} e^{at}$$

X_a and X_b are "reduced" coords.
incorporating natural equilibrium
displacement of the springs.

(2) 

$$\begin{aligned} m x_a'' &= -k x_a + k(x_b - x_a) \\ m x_b'' &= -k(x_b - x_a) \end{aligned} \;\Bigg|\; \begin{aligned} v_a &= x_a' \\ v_b &= x_b' \end{aligned}, \quad \begin{pmatrix} x_a \\ v_a \\ x_b \\ v_b \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-2k}{m} & 0 & \frac{k}{m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m} & 0 & \frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_a \\ v_a \\ x_b \\ v_b \end{bmatrix}$$

$T_r = 0$, $\det = \left(\frac{k}{m}\right)^2$, $P_A(\lambda) = \lambda^4 + 3\left(\frac{k}{m}\right)\lambda^2 + \left(\frac{k}{m}\right)^2$, $\qquad \omega^2 = \left(\frac{k}{m}\right)$, then

$\lambda = \pm i\sqrt{\frac{3+\sqrt{s}}{2}}\,\omega,\ \pm i\sqrt{\frac{3-\sqrt{s}}{2}}\,\omega$. If $\lambda^2 = -\left(\frac{3 \pm \sqrt{s}}{2}\right)\frac{k}{m}$, then eigenvector is $\begin{bmatrix} 2 \\ 2\lambda \\ 1 \pm \sqrt{s} \\ (1 \pm \sqrt{s}\lambda) \end{bmatrix}$.

<u>Interesting result</u>: For the "normal modes" x_b and x_a are either in phase or 180° out of phase.