18.034, Honors Differential Equations
Prof. Jason Starr
Lecture 5
2/13/04

1. Quickly reviewed the proof of existence/uniqueness on a small interval, $[t_0, t_{0+C}]$

2. Explained how to do the same for $[t_{0-c}, t_0]$, and then patch the 2 solutions. Checked the solution is diff. at t_o.

3. Explained how uniqueness on small open intervals implies uniqueness on all open intervals.

4. Defined what a "solution in D" is. We did this before in lecture, but I thought it was best to review it.

5. Used uniqueness on arbitrary intervals to prove that there is an interval (a,b) and a "solution in D" to the IVP $\begin{cases} y' = f(t,y) \\ y(t_0) = y_0 \end{cases}$ such that for every other solution in D, $((a_1,b_1),y_1)$, then $(a_1, b_1) \subset (a, b)$ and y_1 = restriction of y to (a_1,b_1). (A "solution in D" is a solution whose graph lies in the interior of D).

6. Stated the maximal extension theorem: let $(a,b) \cup y$ be as above. Then $y(a) := \lim_{t \to a^+} y(t)$ and $y(b) := \lim_{t \to b^-} y(t)$ exists, and (a,y(a)), (b,y(b)) are boundary points of D. (Here D is a closed region $\left(D = \overline{IVD}\right)$ contained in R). Proof on Tuesday.