18.034, Honors Differential Equations
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Lecture 16
3/10/04

0. Drew pictures of phase portraits of $y'' + ay' + by = 0$ for diff region of (a,b)-plane.
1. Quickly reviewed method of undet. coeff's for
   $p(D)\overline{y} = e^{\lambda t}h(t)$, where h(t) a poly of degrad and $p(\lambda) \neq 0$.

   <u>Example</u>: $y'' + 2by' + \omega_0^2 = F_0 \cos(\beta t)$.

   <u>Homog. eq'n</u>: $\begin{cases} C_1e^{-r_1t} + C_2e^{-r_2t} \text{ if } b^2 > \omega_0^2, \; -r_1, -r_2 = -b \pm \sqrt{b^2 - \omega^2} \\ C_1e^{-bt} + C_2te^{-bt} \text{ if } b^2 = \omega_0^2 \\ C_1e^{-bt}\cos(\omega t) + C_2e^{-bt}\sin(\omega t) \quad \text{if } b^2 < \omega_0^2 \end{cases}$

   $\omega^2 > \omega_0^2 - b^2$. Correspond to overdamped, critically damped, underdamped.

   <u>Inhomog. Eq'n</u>: $b > 0$. $\overline{y}'' + 2by + \omega_0^2 y = F_0 e^{i\beta t}$, $\overline{y} = Ae^{i\beta t}$

   $$\rightsquigarrow A = F_0 \frac{1}{\sqrt{\left(\omega_0^2 - \beta^2\right)^2 + 4\beta^2 b^2}} e^{i(\beta t - \Phi)}, \; \tan(\Phi) = \frac{2\beta b}{\omega_0^2 - \beta^2}.$$

   In particular, for b, ω_0 fixed, the amplitude gain is maximum at "near resonant" frequency

   $$\omega_1 = \sqrt{\omega_0^2 - 2\beta^2} \quad (< \omega^2).$$

2. Quickly discussed method of undet. coeff. if $p(\lambda) = 0$. $p(z) = q(z)(z - \lambda)^l$. So $p(D)e^{\lambda t}g(t) = e^{\lambda t}C(D + \lambda)D^l g(t)$. Write $g(t)$ = general poly. of degree $\partial + l$ (with no terms of degree < l if you like), and then iteratively solve the resulting linear equations for the coeffs.
   Example: $y'' + \omega_0^2 y = F_0 \cos(\omega_0 t)$.

3. Method of undet. coeffs. Derived the formula for a particular sol'n of $Ly = f(t)$:

   $$y_\partial(t) = \int_{t_0}^{t} \left(\frac{y_1(s)y_2(t) - y_1(t)y_2(s)}{W[y_1 - y_2](s)} \right) f(s)ds.$$