Least Squares Interpolation

1. Use the method of least squares to fit a line to the four data points

$$(0,0), \quad (1,2), \quad (2,1), \quad (3,4).$$

<u>Answer:</u> We are looking for the line $y = ax + b$ that best models the data. The deviation of a data point (x_i, y_i) from the model is

$$y_i - (ax_i + b).$$

The sum of the squares of the deviation is

$$\begin{aligned} D &= (0 - (a \cdot 0 + b))^2 + (2 - (a \cdot 1 + b))^2 + (1 - (a \cdot 2 + b))^2 + (4 - (a \cdot 3 + b))^2 \\ &= b^2 + (2 - a - b)^2 + (1 - 2a - b)^2 + (4 - 3a - b)^2. \end{aligned}$$

Taking derivatives and setting them to 0 gives

$$\frac{\partial D}{\partial a} = -2(2 - a - b) - 4(1 - 2a - b) - 6(4 - 3a - b) = 0 \ \Rightarrow\ 28a + 12b = 32 \ \Rightarrow\ 14a + 6b = 16$$
$$\frac{\partial D}{\partial b} = 2b - 2(2 - a - b) - 2(1 - 2a - b) - 2(4 - 3a - b) = 0 \ \Rightarrow\ 12a + 8b = 14 \ \Rightarrow\ 6a + 4b = 7.$$

This linear system of two equations in two unknowns is easy to solve. We get

$$a = \frac{11}{10}, \qquad b = \frac{1}{10}.$$

Here is a plot of the problem.