Hi. Welcome back to recitation. In lecture, you've been learning about computing-- rather, not computing, but determining whether series converge or diverge, and different tests for that. In particular, you've learned the integral test.

So here are a couple of series that you haven't seen before. The sum from n equals 2 to infinity-- 2, just so I don't have any funny business here of dividing by 0-- of 1 over n times log of n. And a second series, sum from n equals 2 to infinity of 1 over n times log of n quantity squared. So the question is, do these series converge or diverge? So why you pause the video, take some time to work on this question, come back, and we can work on it together.

Welcome back. Before you left, I gave you a little hint that these might be questions that are amenable to the integral test. One thing we can do-- you know, if you, if I hadn't given you that hint, how could you figure this out? Well, you can look at these integrands. And they don't really look a lot like anything you've seen before. But the associated functions, right-- so this is the associated function, 1 over x log x. This continuous function is a function that we have-- you know, it looks sort of like some things that we've integrated before. So that's one hint for the integral test. Another hint for the integral test is, you just don't know very many tests right now. So it's kind of a small selection of options.

Another thing is that, you know, it's not going to be a nice series. It's not going to have a nice numerical value. This log n thing is behaving badly. You're not going to be able to compute values, exact values, of the partial sums, any nicer than they looked just by writing down that sum.

So OK. So after we've got the idea of the integrals-- that's how we get the idea for the integral test. Now that we've got the idea for the integral test, what do we do?

Well, we know that this series converges if and only if the associated definite integral, the associated improper definite integral converges.

So let's do the first one first. What's the integral associated with it? Well, we take the integrand, and, you know, we-- frequently, we replace the variable n with the variable x, although that doesn't really matter. And so what we do, is we look at the integral of this integrand over the same region. So in this case, from 2 to infinity. And so what we know is that this sum converges if and only if this integral does.

So then, the reason this is a nice thing to do, is that often there are integrals that are easy to compute, while the associated series are hard to compute. So in this case, this is an integral that we have tools to know, to compute with. And in particular, the tool that we have, is that there's a simple little substitution that will work on this series.

So that's the substitution, u equals log x.

So we make the substitution u equals ln of x. Then du is equal to 1 over x times dx. So this is the integral of-- so 1 over x dx, that's du. And so it's 1 over u du.

This is a definite integral, so I also need to change my bounds. So when x is 2, u is ln of 2, although the lower bound doesn't really matter very much when we do the integral test, because you know, if you change it a little bit, that's not going to change the, you know-- as long as you don't move it across a place where the function explodes, all the interesting stuff is whether the function is big as it goes to infinity. So if you move a little round at the bottom, you'll change its numerical value, but you won't change whether it converges or diverges. But in any case, ln of 2. And then when x goes to infinity, ln of x goes to infinity, so the upper bound is also u equals infinity.

And now this is an easy integral. This is just ln of u between-- well, ln 2, which again, really doesn't matter, and infinity. And we see that at the upper bound, we get ln of infinity, which is infinity. So this thing is infinity. So our original series diverges.

OK. So we've applied the integral test here, and we've found that our series diverges. What about this second one? Well, here, we can again apply the integral test, the similar-looking integrand. And in fact, so we get-- so the integral that we want to look at is the integral from 2 to infinity of 1 over x times log of x squared dx. And the same substitution is going to work here. So we're going to use the substitution u equals ln x, du equals 1 over x times dx. And the bounds are going to be the same. ln 2 to infinity.

OK. But what happens when we make this substitution? Well, the 1 over x dx is still the du. But then here, this time, we have 1 over u squared, right? Because we've got an ln of x squared, and u is ln x. So OK. So we get 1 over u squared. So again, this is easy to integrate. So this is 1 over u squared, so that's going to be minus 1 over u when we integrate it between ln 2 and infinity.

OK. So we take the two values here, as u goes to infinity, minus 1 over u goes to 0. So this is 0 minus, and now with the lower bound, it's minus 1 over ln 2, and this is just ln 2.

So this integral converges to a nice finite value, ln of 2, so that means the sum converges as well. All right? And in fact, if you go back and look at the lecture video, you'll see that you can actually bound the value of the sum in terms of this value, ln of 2, the value of the integral, and the first terms of the sum.

I realized that I should have said one thing at the beginning, which is that we didn't check that the hypotheses of the integral test are valid here. So remember that the integral test only applies if this function that you use is a decreasing positive function on the interval in question. And I didn't actually check those conditions. They're easy to see in this case, right? Because for n bigger than 2, n is positive and increasing, and log n is positive and increasing, so the product is positive and increasing, so the 1 over it is positive and decreasing.

So it's easy to check, in this case, that the conditions of the integral test apply. In the-- when you're doing this out in the real world, if you ever want to apply the integral test, that's something you should check yourself before you go and apply it to anything. Make sure that you really have a function to which it does apply.

But then, once you have a function to which it does apply, especially if it's a nice, easy-to-integrate function like this, you can easily apply it. And in this case, we applied it both directions. We saw an integral where the integral diverges, and an integral where the integral converges. And so the corresponding sums, the first one will diverge and the second one will converge. I'll end there.