Example: $\displaystyle\int_{10}^{\infty} \frac{dx}{\sqrt{x^3+3}}$

We could have used a trig substitution to compute $\int_0^\infty \frac{dx}{\sqrt{x^2+10}}$ in the previous example. We can use the limit comparison method to determine whether an integral is finite even if we're unable to find an antiderivative.

For instance, we can't evaluate $\int_{10}^\infty \frac{dx}{\sqrt{x^3+3}}$. But because:

$$\frac{1}{\sqrt{x^3+3}} \cong \frac{1}{\sqrt{x^3}} = \frac{1}{x^{3/2}}$$

we know that:

$$\int_{10}^{\infty} \frac{dx}{\sqrt{x^3+3}} \cong \int_{10}^{\infty} \frac{dx}{x^{3/2}}$$

and so we know that the integral converges to some finite value.