

Hi. Welcome back to recitation. Today we're going to do a nice little problem involving computing the arc length of a curve. So in particular, consider the curve given by the equation y equals x to the $3/2$. So I have here a kind of mediocre sketch of what that curve looks like. You know, it's curving upwards not quite as fast as a parabola would. So I'm interested in the piece of that curve for x between 0 and 4, which I've drawn here. So why don't you take a minute, pause the video, compute the arc length of this curve, come back, and we can compute it together.

All right. Welcome back. Hopefully you had some luck computing this arc length here. So let's set about doing it. So I'm sure you remember that in order to compute arc length, first you have to compute the little piece of arc length ds . And we have a couple of different formulas for that. So then after that, you get an integral, and then hopefully it's an integral you can compute.

So let's remember what ds is. So there are a couple of different ways to remember it. One way, that I like, is to write ds equals the square root of dx squared plus dy squared. So this always reminds me of the Pythagorean theorem, so I find it easy to remember. And then from here, it's not very hard to get the other form, which is, you can divide through by a dx squared inside and multiply by dx outside.

So you can also write this as the square root of 1 plus dy/dx squared dx . And when you write it in this form-- it's, you know, this is the form that you can actually use to integrate it, to actually compute the value in question.

So in our case, we have y as a function of x , right? So we just have to compute dy/dx . So y is x to the $3/2$, so dy/dx is easy to compute, y prime, dy/dx is just $3/2 x$ to the $1/2$, or $3/2$ square root of x . So ds , then-- well, we just have to plug it in there. So that means ds is equal to the square root of 1 plus-- OK. So now you have to square this. Well, $3/2$ squared is just $9/4$, and the square root of x squared is x . So this is $9/4 x dx$.

So this is the thing that we want to integrate. And now you need bounds of integration. So in our case, this is dx . We want to integrate with respect to x , so we need bounds on x . And luckily we have them. We have 0 less than or equal to x , less than or equal to 4 , the bounds that we want. So the arc length in question is the integral from 0 to 4 of square root of 1 plus $9/4 x dx$.

Now, this curve has the property that this is an integral we actually know how to compute. Right? There's a-- well, OK. So I always lose track of my constants when I do this, so I'm going to do an extra substitution, and then it'll be really easy. But you know, this is an integral-- many of you can probably do this one in your heads, basically, at this point.

This is unusual. Even most polynomials that you write down, computing their arc length is really hard. You get nasty things popping up. So, you know, I sort of conspired to choose a one that will have a value that we can

integrate by hand. You don't need to resort to any sort of numerical method. But it happens, in this case, that that did happen, and that's nice. So we can we can actually write down what this arc length is.

So I'm going to do the substitution, u equals $1 + \frac{9}{4}x$. So with this substitution, I get that du is equal to $\frac{9}{4}dx$, and since I want to substitute it the other way, I could write that as dx equals $\frac{4}{9}du$. And I also need to change bounds, so when x equals 0, that goes to u , I put the 0 here, u is equal to 1 when x is equal to 4. So I put 4 in here. That goes to u equals 10, and so, OK.

With those substitutions, I get that the arc length that I'm interested in is the integral from 1 to 10 of $\frac{4}{9}$ times the square root of u du . OK. And so now this is, you know, really easy. So this is u to the $\frac{1}{2}$, so I integrate that, so I'm going to get u to the $\frac{3}{2}$ divided by $\frac{3}{2}$. So this is $\frac{4}{9}$ times u to the $\frac{3}{2}$ divided by $\frac{3}{2}$ between u equals 1 and u equals 10.

OK. So I can divide here, so this becomes $\frac{8}{27}$ is the constant. So this is 8 over 27 times 10 to the $\frac{3}{2}$ minus 1 to the $\frac{3}{2}$, is just 1.

OK. So now if you wanted to, you know, get a decimal approximation for this number, you could put this into a calculator. You can also kind of eyeball what this is, because 10, the square root of 10 is just a little bigger than 3, so this is, you know, bigger than 27, so this is bigger than 26. So this whole thing is probably about 8 or a little bit larger. Probably going to be a little bit larger than 8, would be my guess. So that's, you know, just rough eyeballing. Since you're all sitting in front of a computer, I'm sure you can get a more precise estimate on your own.

But there we go. So very much just applying the sort of straightforward tools that we've developed for computing arc lengths. You know, using our formulas for the little element of arc length, for the differential of arc length. Computing a derivative, plugging it in. And it happens, in this case, that we got something that we can actually evaluate the resulting integral in a nice closed form.

So I'll stop there.