JOEL LEWIS: Hi. Welcome back to recitation.

You've been doing some work on trig integration. I have a nice example here of a problem that requires trig integration in order to solve. So what I'd like you to do is to compute the volume of the solid that you get when you take one hump of the curve y equals sine a*x and you revolve it around the x-axis. So you take the curve between two consecutive roots, and then you, you know, revolve that around the x-axis and that gives you some, I don't know, vaguely football-shaped thing. And so then the question is, what's the volume of that solid?

So why don't you pause the video, take a little while to work that out, come back, and we can work it out together.

Welcome back.

In order to solve this problem, we just are going to apply our usual methods for computing a volume of a solid of rotation. So in order to do that, remember that one of the things you need is you need to know the region over which you're integrating, and you need to choose a method of integration.

So in this case, looking at this region, here, I'm rotating it around the x-axis. It looks to me-- so we have two choices. We could do shells with horizontal rectangles, or we could do disks with vertical rectangles. Looks to me like vertical rectangles are going to be the way to go for this function. Nice, simple, have their base on the x-axis. You know, this is a nice setup for disks.

So we're going to take vertical disks here, like this. Vertical rectangles that are going to spin into vertical disks. And so we're going to integrate all these disks, we're going to add them up starting at x equals 0, and going until the end of this region. So we need to figure out what the end of that region is, so we need sine a*x to be 0, again. Well, the first 0, the first time sine is 0 after 0 is at pi, so we need a*x to be pi. So this value is at pi over a. OK. So that's the setup for the problem.

And now we just need to remember, you know, how to do a problem like this. So we have each of these disks. Well, its height here is the height of the function, which is y, in this case.

So the area of a little disk-- sorry. The area of a disk, the little-- oh, dear.

The element of volume, the little bit of volume that we get is equal to-- well, the area of the disk is pi y squared, which is pi sine squared of a*x. And then the thickness of the disk is a little dx. So this is our little element dV of volume. And now to get the whole volume, we just integrate this over the appropriate range. So this means V is equal to the integral from 0 to, as we said, to pi over a of pi times sine squared of a*x dx.

So this is just like the sorts of integrals you were doing in class. It's a definite integral. I guess most of the ones you did were just anti-derivatives, but of course, that's an easy translation to make by via the fundamental theorem.

So, OK, so we have this a*x here. You know, it's up to you. I think my life will be simpler if I just make a little u-substitution, get rid of the a*x, and then I don't have to think about it very much anymore. So I can take u equals a*x, so du equals a*dx, or dx equals 1 over a du.

So I, OK, so I make this quick substitution. When x is 0, u is also 0. When x is pi over a, u is just pi. So this becomes the integral from 0 to pi-- I can pull this pi out front. And I can pull-- so I'm going to get pi times sine squared u times dx is 1 over a du. So I'm going to pull the 1 over a out front, as well. So it's pi over a times the integral from 0 to pi of sine squared u du.

OK. So now we've just simplified it to the situation where we've just got a trig integral, no other complications at all. How do we do this one? Well, OK, so this is not one of those nice ones where we have an odd power for either sine or cosine. We have sine is appearing to the even power of 2, and cosine, well, it doesn't appear. It appears to the even power 0, if you like. You could say it that way.

So when we have a situation where sine and cosine both appear in even powers, what we need to do is we need to use one of our trig identities. We need to use a half angle identity. So the identity in particular that we want to use is we want to replace sine squared u with something like a cosine of 2u somehow.

So in order to do that, we need to remember the appropriate identity. So one of the double angle identities is cosine of 2t equals 1 minus 2 sine squared t, which we can rewrite as the half angle identity sine squared t equals 1 minus cosine 2t over 2. So this is true for any value t. In particular, it's true when t is equal to u.

Back over here. So we can rewrite this by replacing sine squared u with 1 minus cosine of 2u divided by 2. So this integral becomes, so our integral is equal to-- well, we've still got the pi over a in the front-- integral from 0 to pi of 1 minus cosine of 2u over 2 du.

OK, and so now we integrate it. So 1 over 2, that's easy. That's just-- so, OK, so the pi over a is still out in front. 1 over 2 integrates, just gives us u over 2. How about cosine of 2u? Well, so minus cosine of 2u, so that should give us a minus sine. Right? Derivative of sine is cosine, derivative of minus sine is minus cosine. So its minus sine of 2u, and then because it's 2u, we're going to have to divide by 2 again. So over 4.

All right. If you don't believe me, of course, you could always check by differentiating this expression and making sure that it matches that expression, the integrand, here. And OK, and so we need to take that between u equals 0 and u equals pi. So luckily we changed our bounds of integration and we don't have to go all the way back to x again.

OK, so this is pi over a times-- OK, so u over 2 minus sine, 2u over 4 when u is pi over 2 minus sine of 2 pi. That's just 0, right? Yes, that's just 0. Good. So this term is just 0. pi over 2 minus-- OK, now when we put in 0, here, well, we get 0 minus sine of 0, so that's just 0-- so just pi over 2.

OK. So the answer, then, is pi squared over 2a. So that's the volume we were looking for.

So just to quickly recap, we have our solid of revolution here that we get by spinning this region around the x-axis. We use our typical methods for computing volumes of solids of revolution. We've got a, when we do that, the integral that we set up is a trig integral with a sine squared in it. So both sine and cosine appear to an even power in this trig integral. When you're in that situation, you're going to have to use your half angle formulas, like so.

Once you do that, you'll simplify. Sometimes life is hard, you'll have to do it more than once. In this case, we only had to do that once, so we got-- then we-- that reduces the integral to something that's easy to compute, where you have one of sine or cosine always appearing to an odd power. In this case, very simple. You just had a cosine. And then you integrate it, and this was our final answer.

I'll end there.