PROFESSOR: Welcome back to recitation. Today in this video, what I'd like us to do is work on some solid of revolution problems. Well, one solid of revolution problem. So what I want us to do in this one is, find the volume of the solid generated by rotating the region bounded by these three curves, x equals 0, y equals 5, and y equals x squared plus 1, about the y-axis. And I'm forcing you-- or I'm requesting-- that you use the disk method. So I want us to work on the disk method in this case. So why don't you take a crack at the problem? And then I'll be back.

OK, welcome back. So what we're going to do is we're going to work on finding the volume of a solid generated by rotating this region that's bounded by the three curves mentioned about the y-axis. And as I said before, we're going to use the disk method. So when you're solving these problems, the first thing you want to do is give yourself at least a rough sketch of what this region looks like, of with the region bounded by these three curves actually looks like.

So the first thing I'm going to do before I do anything else, is at least get a rough sketch of what that region looks like. And I'll do that over here on the right. So, let's see. We know we've got the region-- let me actually even use some different colored chalk. We'll say the x equals 0 is actually the y-axis. That's x equals 0. And then y equals 5-- actually we'll put that in last. But y equals x squared plus 1 is the parabola y equals x squared shifted up by 1. So if this is y equals 1, then it's just roughly something like this. That might not look like the best parabola I've ever seen, but well. Something roughly like that. Maybe, as I said, rough sketch. Maybe more like that.

And then, we want to also put in the y equals 5, which I'll say is roughly about here. So I could-- notice I've got these three curves. This curve, this curve, this curve. So I'm interested in this region right here. Right? Notice that if I also include this region, when I rotated that around, I'd actually get the same solid, whether I included this region or not. But in terms of doing the integration, I want to make sure I don't count anything doubly. So I'm just looking at what happens when I take this piece and I rotate it around the y-axis.

So what do we want to do? We said we want to use the disk method. Which means ultimately, I'm looking for finding out what the radius is. Right? I'm doing pi r squared when I integrate. So I'm interested in finding this length-- that's going to be our radius-- and then we rotate-- you probably can't see that, so let me just say that's going to represent our radius. And then when we rotate that around the y-axis we get our disk. Right? We've seen some pictures of this

before. So I'm interested in this length here.

And because I want to use the disk method, I'm forcing us to figure out what is this value in terms of y? So we know we have, well we know that y equals x squared plus 1 there. So in terms of y, this is y minus 1 equals x squared. So the square root of y minus 1 is equal to x. So this is the expression I'm interested in as our radius, as my radius. Right?

And now, what I need to do is figure out, I know if this is the radius I need to figure out, since I'm doing this in terms of y, what this lower bound is for y and what the upper bound is for y. And then I'm going to be integrating pi times the radius squared over y, over the changes in y.

Now let's see. This one's easy. y equals 5 up here. And what is this value here? Well, the function that I had here was y equals x squared plus 1. So when x is 0, y is 1. So this is the value where y equals 1. So I know what I'm interested in doing. Again, let me write it down. Is I'm integrating from 1 to 5 of pi r squared dy. And now, I need to write r as a function of y.

So r is square root of y minus 1. When I square that, I just get y minus 1. So what I'm really interested in integrating is, the integral from 1 to 5 of-- let me put the pi outside, because who cares-- pi times that integral, y minus 1 dy. And if I actually evaluate this, figure out, if I actually take the integral, use my rules for polynomials, I can get a numerical value. I'm going to stop evaluating here, because from here on I know you guys can do it.

But, the point of this problem, what I want to mention, the point of this problem is the main thing we had to do is figure out what is the radius in terms of y. That was the main thing we had to figure out in order to do this problem. I said you had to use disks. So we had to figure out what the radius was in terms of y. And then we had to figure out what the bounds were in terms of y. And once we do that, it's a simple matter of evaluating this integral, or finding the actual value for what this integral is. So I think that is where I'll stop.