More New Functions from Old

Using definite integrals, we can define many transcendental or "new" functions which cannot be expressed in elementary terms.

Example: Fresnel Integrals

$$C(x) = \int_0^x \cos(t^2)\,dt$$

$$S(x) = \int_0^x \sin(t^2)\,dt$$

These are named after Augustin-Jean Fresnel and are used in optics.

Example: From Fourier Analysis

$$H(x) = \int_0^x \frac{\sin t}{t}\,dt$$

Example: Logarithmic Integral

$$\mathrm{Li}(x) = \int_2^x \frac{dt}{\ln t}$$

This function is significant because it is approximately equal to the number of primes less than x. If you can describe precisely how close the value of $\mathrm{Li}(x)$ is to the exact number of primes less than x you'll have proven the Riemann hypothesis; a task mathematicians have been working on for over a century.

Question: Are we supposed to understand this stuff?

Answer: That's a good question. We're going to see a lot more of the function $F(x) = \int_0^x e^{-t^2}\,dt$ because it's associated with the normal distribution. The functions described in this segment are simply examples of other transcendental functions that are important and described by definite integrals. For this class, the only thing that you'll need to do with such functions are things like understanding the derivative, the second derivative, and sketching their graphs.