Integral of $\sin(x) + \cos(x)$

Consider the following integral:

$$\int_0^\pi \sin(x) + \cos(x)\,dx.$$

a) Use what you have learned about definite integrals to guess the value of this integral.

b) Find antiderivatives of $\cos(x)$ and $\sin(x)$. Check your work.

c) Use the addition property of integrals to compute the value of:

$$\int_0^\pi \sin(x) + \cos(x)\,dx.$$

Check your work by comparing to your answer from part a.

Solution

a) Use what you have learned about definite integrals to guess the value of this integral.

The addition property of integrals tells us that:

$$\int_0^\pi \sin(x) + \cos(x)\,dx = \int_0^\pi \sin(x)\,dx + \int_0^\pi \cos(x)\,dx.$$

We saw in lecture that $\int_0^\pi \sin(x)\,dx = 2$.

The value of $\int_0^\pi \cos(x)\,dx$ equals the (signed) area between the graph of $y = \cos(x)$ and the x-axis. Between 0 and π the amount of area above the axis equals the amount below the axis, so $\int_0^\pi \cos(x)\,dx = 0$

We conclude that:

$$\int_0^\pi \sin(x) + \cos(x)\,dx = 2.$$

b) Find antiderivatives of $\sin(x)$ and $\cos(x)$. Check your work.

The derivative of $\sin x$ is $\cos x$, so $\sin(x)$ is an antiderivative of $\cos(x)$.

The derivative of $\cos x$ is $-\sin x$. To find a function whose derivative is $\sin(x)$ we multiply by -1 to get $-\cos(x)$.

Check your work:

$$\frac{d}{dx}(-\cos(x)) = -(-\sin(x)) = \sin(x);$$

$$\frac{d}{dx}\sin(x) = \cos(x).$$

c) Use the addition property of integrals to compute the value of

$$\int_0^\pi \sin(x) + \cos(x)\,dx.$$

Check your work by comparing to your answer from part a.

$$\begin{aligned}
\int_0^\pi \sin(x) + \cos(x)\,dx &= \int_0^\pi \sin(x)\,dx + \int_0^\pi \cos(x)\,dx \qquad \text{(addition property)} \\
&= -\cos(x)\big|_0^\pi + \sin(x)\big|_0^\pi \qquad \text{(FFT2)} \\
&= [-\cos(\pi) - (-\cos(0))] + [\sin(\pi) - \sin(0)] \\
&= [-(-1) + 1] + [0 - 0] \\
&= 2.
\end{aligned}$$

This agrees with our answer to part a.