

## Riemann Sums

We haven't yet finished with approximating the area under a curve using sums of areas of rectangles, but we won't use any more elaborate geometric arguments to compute those sums.

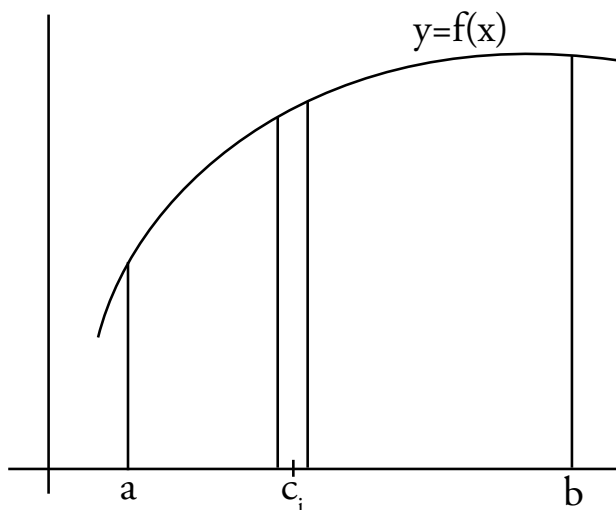


Figure 1: Area under a curve

The general procedure for computing the definite integral  $\int_a^b f(x) dx$  is:

- Divide  $[a, b]$  into  $n$  equal pieces of length  $\Delta x = \frac{b-a}{n}$ .
- Pick *any* value  $c_i$  in the  $i^{\text{th}}$  interval and use  $f(c_i)$  as the height of the rectangle.
- Sum the areas of the rectangles:

$$f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_n)\Delta x = \sum_{i=1}^n f(c_i)\Delta x$$

The sum  $\sum_{i=1}^n f(c_i)\Delta x$  is called a *Riemann Sum*.

This notation is supposed to be reminiscent of Leibnitz' notation. In the limit as  $n$  goes to infinity, this sum approaches the value of the definite integral:

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(c_i)\Delta x = \int_a^b f(x) dx$$

Which is the area under the curve  $y = f(x)$  above  $[a, b]$ .

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