PROFESSOR: Welcome back to recitation. In this video I want us to practice using Newton's Method to find the solution to an equation.

So what we're going to do in particular is we're going to use Newton's Method to approximate a solution to the following equation, 2 cosine x equals 3x. And I'm going to tell you where to start. We're going to the have our initial value, x_0, be pi over 6. And I want you to find x_2. So why don't you pause the video, take a little time to work on that, and then I'll come back and I will show you how I did it.

OK. Welcome back. Again, what we're going to do is use Newton's Method to approximate a solution to this equation. And so what I want to point out first is, I want to point out why pi over 6 is a reasonable first value to choose, and I want to point out that this, in fact, has only one solution.

So what I'm going to do, to give us a reason for that, is I'm going to draw a rough sketch of two curves and show where they intersect. And so I want us to notice that if I were to look at the two curves, y equals cosine x and y equals 3/2 x and I draw them on the same xy-plane, that where they intersect will be where I have solutions to this equation. And that's because I just divide both sides by 2. Whatever solves this equation solves the equation, cosine x equals 3/2 x.

So let me give you a rough sketch of those two curves and we'll see what the intersections look like. So I'm going to do that right down here. OK.

So let me, let me first draw-- make this y equals 1. Make this y equals minus 1. And I'm going to draw cosine x first, y equals cosine x first, because I'm most likely to have a hard time with that, and I'll do my x scale once I'm done. And so cosine x, y equals cosine x, looks something like this. Maybe not the most perfect, but again, it's kind of a rough sketch. That's pretty good. Something like this. So this is y equals cosine x.

And now I want to graph y equals 3x over 2. And that goes through the point (0, 0). It also goes through the point one comma three halves. Well, this is pi over 2 right here. So 1 is about here, we'll say. Because pi over 2 is a little bigger than one and a half. So the 1 is about here, One and a half is about here. Or 3/2, if you need to remind yourself. So the line y equals 3/2 x looks something like this.

So it's fairly straightforward to see that these two curves intersect at one spot, whatever this spot is. OK? And notice, to the left they don't intersect. So we are just looking for a single solution. And then the other thing I want to point out is, why is pi over six potentially a good guess to start with? Well, the value, this is the x-value 1, and this is the x-value 0. We know for a fact that we have to have this x-value line between 0 and 1 because of where my point is at the time that x equal 1, I'm all the way up at y equals 3/2 up here. So at least we know we're between 0 and 1. And then from there you could even try some other values like pi over 3 and pi over 4 and put those in and see how they compared. But at least, we'll just say, at least we know x is between 0 and 1, and pi over 6 is certainly in that region. So that's a good first starting point.

Now I'm going to come over here and start to do some work.

If we want to solve the equation, 2 cosine x equals 3x, what we're really doing is we're looking for zeros of this function. So we find the zeros of this function, which we know there's only one of them. We find the zero of this function, then we actually have solved 2 cosine x equals 3x. So hopefully that makes sense to you that we're actually going to apply Newton's Method to this function. And so when we apply Newton's Method, we need the function. We also need to the derivative. So let me remind you, to derivative of this is going to be negative 2 sine x minus 3. Right? The derivative of cosine x is negative sine x. And so this is exactly the derivative.

And then let me remind you what Newton's Method says. It says the next x-value is equal to the previous x-value minus the fraction of the function evaluated at the previous value divided by the derivative evaluated at the previous value. Right? So this is the formula you have for Newton's Method. So let's see if we can get from x_0 to x_1 and then x_1 to x_2.

So in our case, we have x_1 equals, well, x_0 is pi over 6. And then we have minus the function evaluated at pi over 6, and then the derivative evaluated at pi over 6. So the function evaluated at pi over 6-- cosine of pi over 6 is root 3 over 2. So root 3 over 2 times 2-- we get a root 3. Separate that out. And then here, pi over 6 times 3 is pi over 2. So we get a minus pi over 2. Sine pi over 6 is 1/2. So we get negative 2 times 1/2-- we get negative 1 and then a negative 3.

And if you simplify this, you get that this is approximately 0.564, or around that. OK?

And now from here, you would then, for x_2, you're going to take 0.564 minus these things evaluated at 0.564. This ratio, f of 0.564 divided by f prime at 0.564. But I'm not going to do that because you should get somewhere around, depending on how many decimal places you kept, you should get something around one of these two values.

So you actually get, after x_0, by x_1, you have something that is at least fixed to the first two decimal places. And then this third decimal place, maybe it's going to be a 4 or a 3 in the end. But depending on what value we choose here, we might get slightly different values here based on the rounding. So just suffice it to say, I got x_1. Your x_2 should be about the same. It should be one of these two.

OK. So let me just remind you what we were doing here. We were trying to use Newton's Method to find a solution to an equation that I had written up here, this 2 cosine x equals 3x, and I pointed out a couple things. I pointed out that finding a solution to this equation is the same as finding a solution to the equation cosine x equals 3/2 x. And so I did that as a graph to sort of see if I could get an initial idea of what kind of solution I was looking for.

And then we just started using Newton's Method on a particular function. And that function was this side of the equation minus this side. Because if 2 cosine x minus 3x equals 0, then 2 cosine x equals 3x. So we had this function over here, 2 cosine x minus 3x, and I said I was looking for zeros of that function. And that's where Newton's Method comes in.

So I think that is where I will stop.