Curves are Hard, Lines are Easy

How are linear approximations used? We'll start with an example, then discuss.

Suppose I want to know the value of $\ln(1.1)$. If I've memorized the formula $\ln(1+x) = x$ for $x \approx 0$ I know right away that $\ln(1.1) \approx 0.1$ just by plugging in $x = 0.1$. (This works because 0.1 is "close enough" to zero.)

So what? The value of $\ln(1.1)$ is hard to compute; the value of 0.1 is easy. We used linear approximation to make a "hard" value "easy" to understand.

In general, $f(x)$ is hard to compute and $f(x_0) + f'(x_0)(x - x_0)$ is easy to compute (even though $f(x_0) + f'(x_0)(x - x_0)$ looks uglier). Look at the list below; evaluating the expressions on the left is hard and evaluating the ones on the right is easy.

$$\begin{aligned} \sin x &\approx x \\ \cos x &\approx 1 \\ e^x &\approx 1 + x \\ \ln(1+x) &\approx x \\ (1+x)^r &\approx 1 + rx \end{aligned}$$

That's the main advantage of linear approximation: it lets you work with expressions that are much easier to compute, which lets you make faster progress in solving problems.