$\dfrac{d}{dx}a^x$, part 2

We're learning to differentiate *any* exponential a^x. This is the second of two possible methods.

Method 2: Logarithmic differentiation

It turns out that sometimes it is hard to differentiate a function u and easier to differentiate $\ln u$ (for example, $u = e^{x^2+6}$.) We'd like to be able to use $\frac{d}{dx}\ln u$ to find $\frac{d}{dx}u$.

The chain rule tells us that $\frac{d}{dx}\ln u = \frac{d\ln u}{du}\frac{du}{dx}$, and we know that $\frac{d}{du}\ln u = \frac{1}{u}\frac{du}{dx}$, so

$$(\ln u)' = u'/u.$$

How does this help us compute $\frac{d}{dx}a^x$?

$$\begin{aligned} u &= a^x \\ \ln u &= \ln(a^x) \\ \ln u &= x\ln a \end{aligned}$$

This is pretty easy to differentiate because $\ln a$ is a constant:

$$(\ln u)' = \ln a.$$

Since $(\ln u)' = u'/u$, $u' = u(\ln u)'$. So $\frac{d}{dx}a^x = a^x\ln a = (\ln a)a^x$.

This uses the same arithmetic as the first method, but we don't have to convert to base e.