1. Compute the following derivatives. (Simplify your answers when possible.)

(a) $f'(x)$ where $f(x) = \dfrac{x}{1-x^2}$

(b) $f'(x)$ where $f(x) = \ln(\cos x) - \dfrac{1}{2}\sin^2(x)$

(c) $f^{(5)}(x)$, the fifth derivative of f, where $f(x) = xe^x$

2. Find the equation of the tangent line to the "astroid" curve defined implicitly by the equation

$$x^{2/3} + y^{2/3} = 4$$

at the point $(-\sqrt{27}, 1)$.

3. A particle is moving along a vertical axis so that its position y (in meters) at time t (in seconds) is given by the equation

$$y(t) = t^3 - 3t + 3, \quad t \geq 0.$$

Determine the total distance traveled by the particle in the first three seconds.

4. State the product rule for the derivative of a pair of differentiable functions f and g using your favorite notation. Then use the DEFINITION of the derivative to prove the product rule. Briefly justify your reasoning at each step.

5. Does there exist a set of real numbers a, b and c for which the function

$$f(x) = \begin{cases} \tan^{-1}(x) & x \leq 0 \\ ax^2 + bx + c, & 0 < x < 2 \\ x^3 - \frac{1}{4}x^2 + 5, & x \geq 2 \end{cases}$$

is differentiable (i.e. everywhere differentiable)? Explain why or why not. (Here $\tan^{-1}(x)$ denotes the inverse of the tangent function.)

6. Suppose that f satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ for all real numbers x and y. Suppose further that

$$\lim_{x \to 0} \frac{f(x)}{x} = 1.$$

(a) Find $f(0)$.

(b) Find $f'(0)$.

(c) Find $f'(x)$.