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18.01 Single Variable Calculus  
Fall 2006

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## 18.01 Exam 3

**Problem 1. (20 pts)** Evaluate the following integrals

$$a) \int_0^2 \frac{xdx}{(1+x^2)^2}$$

$$b) \int_{-\pi/2}^{\pi/2} \sin^6 x \cos x dx$$

**Problem 2. (20 pts.)** Find the following approximations to

$$\int_0^{\pi/2} \cos x dx$$

(Do not give a numerical approximation to square roots; leave them alone.)

- Using the upper Riemann sum with two intervals
- Using the trapezoidal rule with two intervals
- Using Simpson's rule with two intervals

**Problem 3. (20 points)** Find the volume of the solid of revolution formed by revolving the  $y$ -axis the region enclosed by

$$y = \cos(x^2)$$

and the  $x$ -axis (central hump, only).

**Problem 4. (20 points)** Students studying for an exam get  $x$  hours of sleep in the two days leading up to the exam, where  $x$  is the range  $0 \leq x \leq a$ . The numbers of students who got between  $x_1$  and  $x_2$  hours of sleep is given by

$$\int_{x_1}^{x_2} cx dx; 0 \leq x_1 \leq x_2 \leq a$$

- What fraction of the student got less than  $a/2$  hours of sleep?
- Their scores are proportional to the amount of sleep they got:  $S(x) = 100(x/a)$ . Find the (correctly weighted) average score in the class.

**Problem 5.** (20 points) Let

$$F(x) = \int_0^x \sqrt{t} \sin t dt$$

- a) Find  $F'(x)$  for  $x > 0$  identify the points  $a > 0$   $F'(a) = 0$
- b) Decide whether  $F$  has a local maximum at the smallest critical point  $a > 0$  that you found in part (a) by evaluating  $F''$ .
- c) Say whether  $F(x)$  is positive, negative or zero at each of the following points, and give a reason in each case.
  - i)  $x=0$
  - ii)  $x=\pi$
  - iii)  $x=2\pi$
- d) Use a change of the variable to express  $G(x) = \int_0^x u^2 \sin(u^2) du$  in terms of  $F$ .