18.01 PRACTICE FINAL, FALL 2003

Problem 1 Find the following definite integral using integration by parts.

$$\int_0^{\frac{\pi}{2}} x\sin(x)dx.$$

Problem 2 Find the following antiderivative using integration by parts.

$$\int x\sin^{-1}(x)dx.$$

Problem 3 Use L'Hospital's rule to compute the following limits.

(a) $\lim_{x\to 0} \frac{a^x - b^x}{x}, \quad 0 < a < b.$

(b) $\lim_{x\to 1} \frac{4x^3-5x+1}{\ln x}.$

Problem 4 Determine whether the following improper integral converges or diverges.

$$\int_1^{\infty} e^{-x^2}dx.$$

(Hint: Compare with another function.)

Problem 5 You wish to design a trash can that consists of a base that is a disk of radius r, cylindrical walls of height h and radius r, and the top consists of a hemispherical dome of radius r (there is no disk between the top of the walls and the bottom of the dome; the dome rests on the top of the walls). The surface area of the can is a fixed constant A. What ratio of h to r will give the maximum volume for the can? You may use the fact that the surface area of a hemisphere of radius r is $2\pi r^2$, and the volume of a hemisphere is $\frac{2}{3}\pi r^3$.

Problem 6 A point on the unit circle in the xy-plane moves counterclockwise at a fixed rate of $1\frac{\text{radian}}{\text{second}}$. At the moment when the angle of the point is $\theta = \frac{\pi}{4}$, what is the rate of change of the distance from the particle to the y-axis?

Problem 7 Compute the following integral using a trigonometric substitution. Don't forget to back-substitute.

$$\int \frac{x^2}{\sqrt{1-x^2}}dx.$$

Hint: Recall the half-angle formulas, $\cos^2(\theta) = \frac{1}{2}(1+\cos(2\theta))$, $\sin^2(\theta) = \frac{1}{2}(1-\cos(2\theta))$.

Problem 8 Compute the volume of the solid of revolution obtained by rotating about the x-axis the region in the 1^{st} quadrant of the xy-plane bounded by the axes and the curve $x^4 + r^2y^2 = r^4$.

Problem 9 Compute the area of the surface of revolution obtained by rotating about the y-axis the portion of the lemniscate $r^2 = 2a^2\cos(2\theta)$ in the 1^{st} quadrant, i.e., $0 \leq \theta \leq \frac{\pi}{4}$.

Problem 10 Compute the area of the *lune* that is the region in the 1^{st} and 3^{rd} quadrants contained inside the circle with polar equation $r = 2a\cos(\theta)$ and outside the circle with polar equation $r = a$.

Problem 11 Find the equation of every tangent line to the hyperbola C with equation $y^2 - x^2 = 1$, that contains the point $(0, \frac{1}{2})$.

Date: Fall 2003.

Problem 12 Compute each of the following integrals.

(a) $\int \sec^3(\theta)\tan(\theta)d\theta$.

(b) $\int \frac{x-1}{x(x+1)^2}dx$.

(c) $\int \frac{2x-1}{2x^2-2x+3}dx$.

(d) $\int \sqrt{e^{3x}}dx$.