18.01 Exam 4

Name: _____

Problem 1: _____ /25

Problem 2: _____ /25

Problem 3: _____ /25

Problem 4: _____ /25

Total: _____ /100

Instructions: *Please write your name at the top of every page of the exam.* The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Name: _____ **Problem 1**: _____ /25

Problem 1(25 points) A solid is formed by revolving about the x-axis the region bounded by the x-axis, the line $x = 0$, the line $x = a$, and the curve,

$$y = b \sin\left(\frac{\pi x}{a}\right).$$

Find the volume of the solid.

You may use the half-angle formulas,

$$\begin{cases} \cos^2(\theta/2) &= (1 + \cos(\theta))/2, \\ \sin^2(\theta/2) &= (1 - \cos(\theta))/2 \end{cases}$$

Name: _____ **Problem 2**: _____ /25

Problem 2(25 points) A solid is formed by revolving about the y-axis the region bounded by the x-axis, the line $x = 0$, the line $x = a$, and the curve,

$$y = \frac{ab}{x} - b.$$

Find the volume of the solid.

It is simplest to use the shell method. But you may use the disk method if you prefer.

Name: _____ **Problem 3**: _____ /25

Problem 3(25 points) A surface is formed by revolving about the x-axis the curve,

$$y = x^3, \quad 0 \leq x \leq 1.$$

Since the curve is revolved about the x-axis, the radius of each slice is y. Compute the surface area of the surface.

Name: _____ **Problem 4**: _____ /25

Problem 4(25 points) Sketch the polar curve,

$$r(\theta) = \sin(\theta)\sin(\theta + (\pi/2)), \quad 0 \leq \theta \leq \pi.$$

Take note: the angle θ varies over only $1/2$ of a complete revolution.

In particular, label the following on your graph,

(i) in which quadrant or quadrants the curve is contained,

(ii) the endpoints of the curve,

(iii) the two slopes of the tangent lines at the endpoints of the curve,

(iv) and the angle or angles θ at which $r(\theta)$ is a maximum.