

Prob. 15.5

A $60^\circ/0^\circ/60^\circ$ layup gives an example of what are called "quasi-isotropic" laminates, having equal stiffnesses in the x and y directions, regardless of the laminate orientation. Verify that this is so for two laminate orientations, one having the middle ply fibers oriented along the x axis and the other with the middle ply fibers oriented at 30° from the x axis. Examine the **B** submatrices (Eqn. 15.22) for these cases, and explain how the laminates do *not* act as isotropic materials.

First laminate:

```
1> plate
assign properties for lamina type 1...

enter modulus in fiber direction...
  (enter -1 to stop): 230e9
enter modulus in transverse direction: 6.6e9
enter principal Poisson ratio: .25
enter shear modulus: 4.8e9
enter ply thickness: .13e-3
assign properties for lamina type 2...

enter modulus in fiber direction...
  (enter -1 to stop): -1
define layup sequence, starting at bottom...
  (use negative material set number to stop)

enter material set number for ply number 1: 1
enter ply angle: 60                                     <=== 60/0/-60 layup

enter material set number for ply number 2: 1
enter ply angle: 0

enter material set number for ply number 3: 1
enter ply angle: -60

enter material set number for ply number 4: -1

laminate stiffness matrix:

    0.3576E+08  0.1110E+08  0.0000E+00  0.0000D+00  0.0000D+00 -0.8528D+03
    0.1110E+08  0.3576E+08  0.0000E+00  0.0000D+00  0.0000D+00 -0.2423D+04
    0.0000E+00  0.0000E+00  0.1233E+08 -0.8528D+03 -0.2423D+04  0.0000D+00

    0.0000E+00  0.0000E+00 -0.8528E+03  0.1485D+00  0.1996D+00  0.0000D+00
    0.0000E+00  0.0000E+00 -0.2423E+04  0.1996D+00  0.6402D+00  0.0000D+00
   -0.8528E+03 -0.2423E+04  0.0000E+00  0.0000D+00  0.0000D+00  0.2152D+00

laminate compliance matrix:

    0.3114E-07 -0.5507E-08  0.0000E+00  0.0000D+00  0.0000D+00  0.6140D-04
   -0.5507E-08  0.1188E-06  0.0000E+00  0.0000D+00  0.0000D+00  0.1316D-02
    0.0000E+00  0.0000E+00  0.3276E-06  0.3699D-03  0.1125D-02  0.0000D+00

    0.0000E+00  0.0000E+00  0.3699E-03  0.1201D+02 -0.2345D+01  0.0000D+00
    0.0000E+00  0.0000E+00  0.1125E-02 -0.2345D+01  0.6548D+01  0.0000D+00
    0.6140E-04  0.1316E-02  0.0000E+00  0.0000D+00  0.0000D+00  0.1970D+02
```

Second laminate:

```
2> plate
assign properties for lamina type 1...

enter modulus in fiber direction...
  (enter -1 to stop): 230e9
enter modulus in transverse direction: 6.6e9
```

```

enter principal Poisson ratio: .25
enter shear modulus: 4.8e9
enter ply thickness: .13e-3
assign properties for lamina type 2...

enter modulus in fiber direction...
  (enter -1 to stop): -1
define layup sequence, starting at bottom...
  (use negative material set number to stop)

enter material set number for ply number 1: 1
enter ply angle: 90                                     <=== 90/30/-30 layup

enter material set number for ply number 2: 1
enter ply angle: 30

enter material set number for ply number 3: 1
enter ply angle: -30

enter material set number for ply number 4: -1

laminate stiffness matrix:

    0.3576E+08  0.1110E+08  0.1000E+01   0.2157D+04  0.6798D+03 -0.1211D+04
    0.1110E+08  0.3576E+08  0.3600E+02   0.6798D+03 -0.3516D+04 -0.4264D+03
    0.1000E+01  0.3600E+02  0.1233E+08  -0.1211D+04 -0.4264D+03  0.6798D+03

    0.2157E+04  0.6798E+03 -0.1211E+04   0.3598D+00  0.1113D+00 -0.1575D+00
    0.6798E+03 -0.3516E+04 -0.4264E+03   0.1113D+00  0.6057D+00 -0.5543D-01
   -0.1211E+04 -0.4264E+03  0.6798E+03  -0.1575D+00 -0.5543D-01  0.1268D+00

laminate compliance matrix:

    0.8431E-07 -0.3676E-07 -0.7405E-07  -0.4042D-03 -0.2429D-03  0.4705D-03
   -0.3676E-07  0.1281E-06 -0.1869E-08  -0.1921D-03  0.8391D-03  0.2180D-03
   -0.7405E-07 -0.1869E-08  0.2026E-06   0.7496D-03 -0.2410D-05 -0.8699D-03

   -0.4042E-03 -0.1921E-03  0.7496E-03   0.9979D+01 -0.1680D+01  0.3132D+01
   -0.2429E-03  0.8391E-03 -0.2410E-05  -0.1680D+01  0.7249D+01  0.1597D+01
    0.4705E-03  0.2180E-03 -0.8699E-03   0.3132D+01  0.1597D+01  0.2236D+02

```

Note that the 1,1 and 2,2 elements of the stiffness matrix are the same, and also the same for the second laminate oriented 30° from the first. This is an apparent isotropy: stiffness the same in all directions (though only verified here for four directions). Neither laminate is symmetric, so there are nonzero coupling terms in the **B** submatrices, which is not expected in isotropic materials.

The appearance of isotropy is not evident in the compliance matrix, where the 1,1 and 2,2 elements are different; this is a consequence of the extension-curvature coupling that occurs in the nonsymmetric laminate. If the laminate is made quasi-isotropic and symmetric, as in the 60/0/-60/-60/0/60 layup below, the appearance of isotropy occurs in both the stiffness and compliance matrices.

```

l> plate
assign properties for lamina type 1...

enter modulus in fiber direction...
  (enter -1 to stop): 230e9
enter modulus in transverse direction: 6.6e9
enter principal Poisson ratio: .25
enter shear modulus: 4.8e9
enter ply thickness: .13e-3
assign properties for lamina type 2...

enter modulus in fiber direction...
  (enter -1 to stop): -1
define layup sequence, starting at bottom...
  (use negative material set number to stop)

enter material set number for ply number 1: 1
enter ply angle: 60

```

```
enter material set number for ply number 2: 1
enter ply angle: 0

enter material set number for ply number 3: 1
enter ply angle: -60

enter material set number for ply number 4: 1
enter ply angle: -60

enter material set number for ply number 5: 1
enter ply angle: 0

enter material set number for ply number 6: 1
enter ply angle: 60

enter material set number for ply number 7: -1
```

laminata stiffness matrix:

```
0.7152E+08 0.2221E+08 0.0000E+00 0.2441D-03 -0.1221D-03 0.0000D+00
0.2221E+08 0.7152E+08 0.0000E+00 0.0000D+00 0.0000D+00 0.2441D-03
0.0000E+00 0.0000E+00 0.2466E+08 0.0000D+00 0.0000D+00 0.1221D-03

0.2441E-03 -0.1221E-03 0.0000E+00 0.3017D+01 0.1244D+01 0.6652D+00
0.0000E+00 0.0000E+00 0.2441E-03 0.1244D+01 0.4000D+01 0.1890D+01
0.0000E+00 0.0000E+00 0.1221E-03 0.6652D+00 0.1890D+01 0.1368D+01
```

laminata compliance matrix:

```
0.1547E-07 -0.4804E-08 -0.1010E-22 -0.1685D-11 0.5872D-12 0.8656D-12
-0.4804E-08 0.1547E-07 0.5762E-23 0.7364D-12 0.3160D-11 -0.7484D-11
-0.1349E-23 0.9467E-24 0.4055E-07 0.3092D-12 0.4847D-11 -0.1046D-10

-0.1667E-11 0.1170E-11 0.1193E-11 0.3821D+00 -0.8927D-01 -0.6247D-01
0.3895E-12 -0.2733E-12 -0.2482E-11 -0.8927D-01 0.7403D+00 -0.9792D+00
0.2726E-12 -0.1913E-12 -0.7698E-12 -0.6247D-01 -0.9792D+00 0.2114D+01
```