PROFESSOR: --not of the vindictive sort. You skip class, you've skipped a lot of important stuff. But I'll get you on the quiz, that's all.

Kidding aside, there were a number of things that I passed out. I think nobody needs a set of problem set number 13. That was the one that had something on symmetry constraints and working with second-rank tensors. If anybody missed that, you can see me during break.

Some people, I think, missed problem set 14. And that's the one where you are invited to diagonalize some tensors, either using the method of successive approximations or the direct diagonalization-by-an-eigenvalue procedure. Anybody need one of that? And I'm sure that nobody has a copy of problem set number 15, which deals with piezoelectricity. And I know you don't have it, because I just put it together.

So I'd like to hand it out and invite you to explore things that deal with third-rank tensors. And I hope that, even though doing the problem sets is optional, particularly at this juncture in the semester when things have come to a set of successive crunches. But if you don't know how to do it, for goodness' sakes, come see me.

Or raise it in our next class. You know, some question like, I haven't the foggiest idea how to do problem number two. Could you say a little bit about that, please? And I'd be happy to oblige.

All right. I will have for you next time the quizzes and also all the problem sets which will have been turned in up to that point. And what I spent my time doing instead is writing out notes for those people who missed the last lecture, and also notes

covering what we're going to do today. Because it is very exquisitely intensive, algebraically.

It's not hard, but there are a lot of variables with a lot of subscripts. So let me pass this around. I'll split it up into packs.

These are notes on some basic relations in electromagnetism which you may or may not have forgotten. Take one off the top. And it's coming at you from either side, so you're going to pass it back. And the notes also cover everything that we're going to do on piezoelectricity. Most of it will take place today. And I can zip along a little more rapidly if you have notes to follow.

I would like to ask you when you get a set of the notes-- I could really kick myself-- the introductory discussion reminds you of the definition of a dipole. And down in the middle of the page, on the cover sheet, two different types of polarizability are defined. And one of them involves the separation of charge on an individual atom. And that is called, quite appropriately, the electronic polarizability because it involves polarization of the electrons and protons on the individual atoms.

And then there's another type of induced dipole moment that comes when the structure is ionic. And then an electric field will pull positive ions in one direction and negative ions in the opposite direction. And that is very often referred to as the ionic polarizability. And it's easy to keep them straight.

One involves electrons, which all atoms have. The other involves ions. And not all structures and materials have ions. So the second one is unique to ionic structures.

And then, these are sometimes also referred to as the dielectric polarizability. And I meant to purge that from the notes. And as I put this together to xerox it, I grabbed the uncorrected sheet.

So please, just below you see the displaced positive and negative ion on the middle of the page, cross out "dielectric" polarizability. And change that to "ionic" polarizability. And I didn't catch that. There's one other little typo as we go partway through.

In any case, we'll talk about third-rank tensor properties. We'll introduce some other ones other than piezoelectricity later on. But piezoelectricity is one of the primary examples of a third-rank tensor property. And it's one that has a lot of applications in devices-- pressure sensors, audio equipment, electronic devices. It's a very important property in terms of devices and present-day technology.

But there are others. And we'll cover those in due course. Some of them are rather exotic.

OK. But to review these basic concepts in electromagnetism, we remind you again of the definition of a dipole. We mentioned this last time. But to quickly review, a dipole is a pair of charges of opposite sign but equal magnitude, separated by a separation, d. And then one defines a dipole moment, which has a vector character, as the product of one of the charges of magnitude, q. And the vector that separates the two charges in the sense of the vector is defined as going from the negative charge and pointing towards the positive charge.

It's purely a definition. But the reason it's convenient is that the vector sense and the product of charge and separation comes up again and again in all sorts of problems, among them definition of the piezoelectric effects. Also a dipole in an electric field is going to experience a torque because the electric field will pull on the positive charge in the same direction. It will pull on the negative charge in the opposite direction from the field. And that's going to create a torque on this little gizmo.

OK. Now it's important, too-- and interesting to note-- that there are three different kinds of dipole moments. There are some molecules-- and water is the primary example. Water has seen an asymmetrical arrangement of hydrogen, relative to the oxygen ion to which they are connected. And that gives water a permanent dipole moment, which is what makes water such a darn good solvent. And its ability to dissolve primordial juices probably accounts for our fact, intelligent design notwithstanding, of why we are here today.

On the other hand, dipoles, as I was just saying, can be induced when you impose an electric field on matter. And these are of two kinds. One is the dipole moment that is induced on an individual atom. And that results in displacement of the positive nucleus relative to the negative electron shell.

It's found that the dipole moment is proportional to the magnitude of the electric field. And the proportionality constant, alpha, is called the polarizability. And for an individual atom, as I said a moment ago, it's defined as the electronic polarizability.

We write it as a scalar quantity, but actually, by now you're probably sensitized to being a little bit skeptical when things that relate to vectors are described as a scalar. And in fact, the electronic polarizability is not a scalar, it's a tensor. And one should really write that the i-th component of the dipole moment is given by alpha i,j times the j-th component of the electric field.

Second type of induced dipole moment involves, again as we said a moment ago, the effect of imposing an electric field on an ionic structure. And again, the field will pull the positive ions in one direction and negative ions in the other. And here quite clearly, if this pair of ions is in a structure, and that structure has some symmetry, we really have to consider this second origin to induce dipole moments as a tensor. And this is referred to as the ionic polarizability.

So both of these types of dipole moments will be present in matter, in general. The relative importance of each depends on whether the electric field is a static field or an oscillatory electric field. And then the frequency dependence of these two polarizabilities has a consequence on the magnitude for fields of different frequencies. And not surprisingly, the ability of the ions in the structure to polarize is going to poop out with high-frequency electric fields a lot quicker than just the displacement of the light electrons about a positive nucleus.

So there is a frequency dependence of the net polarizability. We won't go into that. But just keep in mind that at very high-frequency electric fields, the ionic polarizability will damp out.

OK. Then we went through a rather simplified but amusing model for the electronic polarizability. And there's some rather severe assumptions that are made. But making those assumptions let's you get a rigorous result, which tells you something about the electronic polarizability.

So we model the electron distribution on the atom as a uniform charge density in a distribution that goes up to some radius, r, and then quits. So there's a sharp cutoff to the distribution of electrons, which is obviously ridiculous. You know there are a collection of wave functions that give you charge probabilities that tail off slowly to large distances, getting progressively smaller and smaller. So this is not terribly realistic.

And then the other thing we assume to make this model, something that we can solve exactly, is that the nucleus and the electron distribution displace as units. In other words, we start with a sphere of electrons, the center displaces, but it stays a sphere of uniformly distributed electrons.

And then having made those assumptions and having lost any credibility for the model, if we carry through to see what the model predicts, it's rather interesting. We use a fact, again, known to freshman and sophomore students of electromagnetism, that a charge inside of a uniform distribution of charge experiences no force. And that's surprising, but it's something that you are very often invited to do on problem sets.

So therefore, if the center of the electron distribution is displaced from the nucleus, then the restoring force between the electron sphere and the nucleus is simply the coulombic force between a nucleus of charge plus ze, and a fraction of the total number of electrons, namely that fraction of the electrons which are contained within a sphere that has a radius equal to the displacement. And that geometry and that algebra's carried out for you on the bottom of the first page.

If you set that up and ask what the dipole moment will be, it comes out beautifully simple. It comes out to be equal to whatever proportionality constant you use in Coulomb's law. I use rationalized MKS units from force of habit. So there's a 4 pi

epsilon 0 in there, then times the cube of the radius of the electron distribution, times the electric field.

So the two items of note that come out of this simplified treatment is first of all, the induced dipole moment is proportional to the applied electric field, which is what we assumed. And so therefore, the polarizability, which is the quantity that relates the dipole moment to the magnitude of the field, is a constant. And it turns out to be equal to 4 pi epsilon 0, times the radius. So not only does this tell us that the electronic polarizability is something that relates dipole moment in direct proportion to the magnitude of the field. And secondly, the electronic polarizability involves the radius of the charge distribution, cubed.

And even on a qualitative basis, this is interesting. It tells you that high-atomic-number, big, fat ions are going to have a very, very large polarizability. And things way down in the periodic table, like beryllium and lithium, and other low-z atoms, are going to be tough little nuts that don't display much polarization at all. And in point of fact, several individuals have tabulated empirical sets of electronic polarizabilities.

One of the earliest ones are the so-called TKS values published a long time ago by Tessman, Kahn, and "Wild Bill" Shockley. And I give you the reference to those. There is another set of values that were assembled by a fellow at DuPont named Bob Shannon. And I'll give you a citation to those values.

But in any case, if you look at these values, you find that the cation that has highest electronic polarizability is thallium, way down on the bottom of the periodic table, next to lead. And that's just a big, fat, flabby atom that can be deformed very, very easily.

How do you get these polarizabilities? Well, the equations at the bottom of page two-- which we won't make any use of but, nevertheless, will tell you where they come from-- there's a relation between the square of the index of refraction and the sum of the polarizabilities of the individual species, times the number of those species per unit volume. And that is something that's called the Lorentz-Lorenz equation, equation. I can't resist saying everything twice, Lorenz and Lorentz.

You can put this in another form that involves the molecular weight of a molecular structure and the polarizability per molecule. It's the same equation, but for organic compounds. It's a useful form.

And it turns out that the dielectric constant of the material is directly related to the square of the index of refraction. So you can write those two equations in terms of either the dielectric constant or the index of refraction. And if you substitute dielectric constant in place of n squared in those equations, the equations get new names.

And they're not called the Lorentz-Lorenz equation, equations. They're called the Clausius-Mossotti equations, which shows you that sometimes fame and immortality can be gained simply by a trivial substitution of variables. Nice to keep in mind if you can find something like that.

Finally, we don't see dipole moments on individual atoms or molecules. We see evidence of polarization in bulk. And on page three is a little model that reminds you of what the polarization of a material is. And that's defined as simply the dipole moment per unit volume. And that's represented by a capital P rather than a small p.

And this is also a vector quantity. And we can see how this is related to the individual dipole moments in the solid by dividing the solid up into individual cells. And I use the term of "cell" loosely. Is this a unit cell? Is this a box around each of the atoms?

It really doesn't matter. Because all of these little dipoles on the individual unit cells are packed together back to front in the solid. So internally, the negative end of one induced dipole moment is always adjacent to the positive end of the neighboring dipole moment. And everything cancels out internally, except for the two surfaces of the solid. So macroscopically, if you impose an electric field and it induces dipole moments, what happens is you see that there is a charge on the surface of the piece of material.

And that is a real charge. It's a bound charge. You can't draw that charge off as a current. Because the minute you reduce and eliminate the electric field, those dipoles disappear. And the charges all go back to where they came from. So this is the thing that keeps the exact model that we use for one of the cells in the solid [INAUDIBLE] of no importance whatsoever.

You could say that the dipole moment on each atom is the charge times the separation of the positive and negative charge. And qi is the induced charge. The number of cells per unit volume, if we say that they're little cubes of the same edge length, delta, is unit volume one divided by the volume per cell, which will be delta cubed.

And the dipole moment per unit volume is the number of cells per unit volume times the polarization on each. And you find that everything drops out. And the polarization indeed turns out to be equal to induced charge per unit area, whether that's induced charge per unit area of one of our cells or on a square centimeter of material. So polarization, dipole moment per unit volume is numerically equal to, and physically equivalent to, an induced charge per unit area.

OK. So much for freshman electromagnetism in fast forward. And now I'd like to turn to something that involves tensors and materials. As I mentioned a moment ago, piezoelectricity, literally "pressure electricity," is a very technologically important property, useful property, but also provides a nice example of a tensor property that has to be defined in terms of a sensor of third rank.

There are a number of different piezoelectric effects. The first one is the so-called direct piezoelectric effect. And it refers to the fact that if you subject a material to an applied stress, it will develop a surface charge. Remove the applied stress, the surface charge goes away.

The surface charge can be described in terms of a polarization, a dipole moment per unit volume. And it will be in direct proportion to a stress tensor sigma j,k. And so what we'll assume-- and this is an assumption. And it's followed, except for very extreme conditions, that each component of the polarization, p sub i, is given by a

linear combination of every one of the nine elements of stress, sigma i,j.

Oops, not i,j. I have to use a different index. I'm used to writing i,j. So each component of the polarization, where i ranges from 1 to 3, is given by a linear combination of all nine of the elements of stress, sigma j,k. And the proportionality constants, di,j,k, are known as the piezoelectric moduli. And this is called the direct piezoelectric effect.

OK. That's simply an assumption that the polarization is proportional to the applied stress. We know that stress, however, is a field tensor of second rank. We know that the polarization has the character of a vector, charge times the length.

We know how second-rank tensors transform. We know how first-rank tensors transform. And therefore, knowing that, we can say that the coefficients, the array of coefficients, di,j,k, will transform like a third-rank tensor. And therefore they are a tensor.

So we will have three components of polarization. And we will have nine components of stress. And so there will be 27 piezoelectric moduli, di,j,k. So things go up rapidly in terms of number of coefficients as the rank of a tensor increases.

Now let's take a look at the nature of these equations. We'll have, for example, the x1 component of P being given by d1,1,1 times the element of stress sigma 1,1. The next term will be d1,2,2 times sigma 2,2. I'm putting down the tensor components first. But this is just an equation. I can write the terms in any order.

Next will be a d1,3,3 times sigma 3,3. And the next one would be a d1,2,3 times a shear component of stress sigma 2,3 and other terms. I don't want to write out all nine of them. Because I think I have written enough to make my point.

The point is that the value of i is always tied to the component of the resulting vector that we're defining. But the second pair of subscripts, j and k, always go together as an unseparable pair with the component of stress that they modify. So the 1,1 here goes with the 1,1. The 2,2 goes with the 2,2. The 3,3 goes with the 3,3.

So why in the world, if they're always going to go together, do we have to use two indices to define the piezoelectric moduli? Why don't we use a single symbol? Well, there is a good reason for not doing it. But we'll issue that caveat later on.

We could use an a, a b, and a c, or an alpha or a beta or a gamma, or some other esoteric symbol. But what makes sense since we're using Arabic numerals to represent subscripts, let's write the pair of indices in terms of an index that's related to the stress tensor, which is sigma 1,1; sigma 2,2; sigma-- whoops. Sigma 1,2; sigma 1,3.

And then comes sigma 2,1; sigma 2,2; sigma 2,3; sigma 3,1; sigma 3,2; sigma 3,3. We know that this is a symmetric tensor. So these off-diagonal terms are always equal to one another. So we're numerically going to have to enter each of those twice.

So to define a single index that represents the components of stress. And I'll describe it in this fashion so you can remember, as a mnemonic device, what makes this work. We'll go down the main diagonal of the stress tensor, this fashion. And then having reached the bottom, we'll go up along the right-hand side and then jog over to the left to pick up the last term.

And we'll define a number associated with these indices that goes in the form 1 to 2, to 3, to 4, to 5, to 6. So we'll just use these three integers, ranging from 1 to 6, to represent pairs of integers in the stress tensor. So things tidy up quite nicely then. This would be P1 is d1,1 times sigma 1; plus d1,2 times sigma 2; plus d1,3 times sigma 3; plus d1,4 times sigma 4.

And now, uh-oh, Houston. We've got a problem. Because there's another term in here that is d1,3,2 times sigma 3,2. And sigma 3,2 is required, because the stress tensor is symmetric, to be numerically equal to sigma 2,3. So this is really d1,2,3 times sigma 2,3.

And then we've got a d1,3,2 times a sigma 3,2. I know that this is equal to this. But is there any reason d1,2,3 has to be equal to d1,3,2? I can't see any reason why.

OK. Let me confide in you that, yes, they are equal. But we can't show it or claim it on the basis of what we've got before us right now. It's going to come later. But we can show that they are equal.

OK. But equal or not, this means we'll have a term d1,4 sigma 4, and we're going to get it in there twice. So when the subscripts go up to 4, we're going to get a 2 out in front. And then we'll get for the term sigma 1,3 and 3,1 we'll have a d1,5 times a sigma 5. This would be sigma 1,3. And then we'd have another d1,5 for sigma 5 again. And this would be sigma 3,1.

So what are we going to do? We're going to say, well, it's nice we're getting rid of an unneeded subscript. P1 is the d1,1 times sigma 1; plus d1,2 times sigma 2; plus d1,3 times sigma 3. Are we then going to say 2 d1,4 times sigma four, and say that the coefficient here is di,j when i is equal to 1, 2, 3. But the coefficient is 2 di,j when j is 4, 5, or 6? Hell of a matrix that would be. That's going to be a bother.

Since it doesn't seem that we can measure these two coefficients, d1,2,3 and d1,3,2, independently anyhow, let's just write our relation in reduced subscripts with the two added together as the element d1,4. So we are going to define d1,4 equals d1,3,2 plus d1,2,3. So that's a definition.

And d1,5 will be defined as the sum of d1,1,3 plus d1,3,1. And we're going to define d1,6, finally, as d1,1,2 plus d1,2,1. OK. So if we do that, then and only then are we entitled to write a nice, simple, compact little nugget that says that in our redefined form, Pi is equal to di,j times sigma of j. And i goes from 1 to 3, and j goes from 1 to 6. And this is kind of a neat compact, easily managed outcome.

So the moral of this story is, I like to think, is you can have your cake if you eat its 2. Oh come on, this is a tough, tough crowd. You're just all worn out from the MRS meeting. That'll do it to anybody.

So why do we worry about the fact that to the direct piezoelectric effect should be a third-rank tensor? The answer, my friends, is that this is no longer a tensor relationship. It's a matrix relationship. A tensor is a matrix, but it's a matrix with a

difference. It's a matrix for which a law of transformation is defined, if you change axes.

There's no such law defined for the $d_{i,j}$'s. So they qualify as a matrix, but they are not a tensor. So to attempt to say-- as you might be inclined to do when we raise the issue of what the symmetry restrictions are on these tensors-- if you attempt to say, well, I'm going to find $d_{2,1}$ prime. And that's going to be $c_{2,i} c_{1,j}$ times $d_{i,j}$.

Wrong. You go down in flames because that's just nonsense. That's just nonsense.

So this is a matrix relation. And if you want to do exercises such as slice a piezoelectric wafer out of a piece of quartz, and then having done so, refer the properties to axes taken along those edges of the plate that you've cut, you've got to-- in order to find the piezoelectric moduli for that plate in that new coordinate system-- you've got to be prepared always to go back to the full tensor notation.

If you want to derive symmetry restrictions, which we're going to do whether you want to or not-- but we won't do them exhaustively-- you've got to go from this matrix notation back to the full three-subscript tensor notation. OK? Yes, sir?

AUDIENCE: Couldn't you extend c sub i,k's to be [? 3,6's ?] and extend most others [INAUDIBLE]?

PROFESSOR: Oh, yeah. No, if we would write all these down, we'd have-- I don't know if that's what you're asking-- but we'd have P_1 is equal to $d_{1,j}$ times sigma sub j; P_2 is equal to $d_{2,j}$ sigma sub j; and P_3 is equal to $d_{3,j}$ times sigma sub j. So I just did that for the line with i equal to 1, because I was too lazy to write down all three relations. Is that what you're asking?

AUDIENCE: No, I'm saying you could have a transformation model if you were to extend a c sub i,j matrix to be 3,6.

PROFESSOR: No. It just won't do it. I mean, think of what these indices are. They're 1, 2, and 3 standing for the x1 direction, the x2 direction, the x3 direction. What's the x5 direction? It's just not defined. Good try, but you just can't do it.

AUDIENCE: I'll think of something next time.

PROFESSOR: Oh, I'm sure you will. Whether it will work or not is an absolutely different question. OK. So beware. Do not try to transform tensors of third rank expressed with two subscripts to other coordinate systems.

All right. What I would like to do now, then, is to examine the symmetry restrictions that are imposed on a third-rank tensor by crystallographic symmetry. And these are embedded in the notes that you have. But it's going to be bothersome to leaf through those. So I separated out the pages separately. And I'm going to look at a few transformations so that you can see how they work, and then point out some very, very curious consequences of this, which are non-intuitive.

So first of all, how would we do it? If we want to get the form of $d_{i,j}$, the matrix for-- let's do a very, very simple one. Well, let's do one that we did last time, for those who were here, for inversion. For inversion, the direction cosine scheme is $c_{1,1}$, 0; 0, 0-- I'm sorry. For 1 bar, the transformation of axes is $c_{1,1}$, 0, 0; 0, $c_{2,2}$, 0; 0, 0, $c_{3,3}$.

So we'll take $d_{i,j}$, change it to the full three-subscript notation, $d_{i,j,k}$. And the law for transformation of the third-rank tensor is $d_{i,j,k}$ prime is equal to c_i, capital I; c_j, capital J; c_k, capital K; times d, capital I, capital J, capital K. We've not done this much. But we've mentioned quite some time ago that this is the way a third-rank tensor would transform element by element.

The only terms that are non-zero in this array, when the symmetry transformation is inversion-- where x_1, x_2, x_3 goes to minus x_1, minus x_2, minus x_3-- is whatever i is, minus 1; whatever j is-- only the diagonal terms are there, and they're all minus 1-- whatever k is, it's going to be minus 1, times $d_{i,j,k}$.

So we find that for inversion, $d_{i,j,k}$ prime is always, regardless of what the three indices are, is going to be minus $d_{i,j,k}$. And if inversion is a symmetry operation which the crystal possesses, we're demanding that the transform index be identical to the original index. But there's a minus sign in there. So we can say that every

single element vanishes, has to vanish.

So any crystal that has inversion in it is not going to be able to display the piezoelectric effect or any other third-rank tensor property. The electro-optic effect, piezoresistance, and there are a whole slew of them, which are examined for you in part on this sheet. One third-rank tensor property is the direct piezoelectric effect, which we've been discussing as our example. There is something called the converse piezoelectric effect, which describes the phenomenon where an applied electric field creates a strain.

So the direct effect is you [? scush ?] your material, you develop charge. The converse piezoelectric effect says if you apply an electric field that's going to move the atoms around and induce polarizations, you are going to create a strain. And the absolutely mind-boggling thing is that the same array of 3 by 9 coefficients describe both the direct piezoelectric effect and the converse piezoelectric effect.

So in one relation, we have that each component of polarization is di,j,k times the nine elements of stress. And that means we have to write three equations in nine variables, the elements of stress. And the converse piezoelectric effect, we're developing a strain. And there are, therefore, nine elements of strain.

And we're applying a vector, e sub i, a field. So there are three components of that vector. 27 elements, 3 times 9. 27 elements, 9 times 3.

So both of these are third-rank tensors. Strain transforms like a second-rank tensor. Field vectors transform like a first-rank tensor. Therefore, the coefficients are elements of a third-rank tensor. But what boggles the mind is that the same 27 numbers describe these two seemingly disparate phenomena.

You don't prove this by symmetry. You don't prove this by the nature of stress and strain. This hinges on the thermodynamic argument, which I'm not going to go into.

But what you do if you're willing to stretch tensor notation a little bit-- you can't have a 3 by 9 array when you've got a tensor of second rank on the left and a vector on the right. You have to write this in this fashion, that di,j,k times ei gives you the

element of strain, epsilon j,k. And that's not proper tensor notation. But we're not going to quibble. Because if you allow this little departure from convention, then you can write both the converse piezoelectric effect and the direct piezoelectric effect in terms of the same matrix.

But again, that is not intuitively obvious. It does not have to be the case. And it hinges on an argument in thermodynamics.

Some other examples of piezoelectric relations that I've mentioned in the notes. If you apply a stress and you get a polarization, that stress produces a strain. So you also have to get a polarization if you're applying a stress, and write it in terms of the strain that's produced. Similarly, if you apply an electric field and it produces a strain, the crystal must be in a state of stress. So there must be relation between stress and applied electric field, which will also be a third-rank tensor.

Those two relations are represented by coefficients given the symbol e. And again, the same array of 27 elements describes both effects. And these are not dignified with any special names. They're just tensor relations that have to be true because of the fact that stress and strain are coupled by elastic relations. Not surprisingly, then, the coefficients e must somehow involve the piezoelectric moduli $d_{i,j}$ and the elastic properties of the material.

Another effect which is a very interesting one is the electro-optic effect. If you apply an electric field to a material, you change the birefringence of the field. The birefringence is defined as the difference in index of refraction for light polarized in two orthogonal directions.

Another tensor effect-- and I'll give you a note defining the terms next time we meet-- there is a piezoresistive effect, which you don't see talked about very much. But that's a property that Texas Instruments was interested in at one time. And I have a set of sheets defining those relations that the presenter of a paper at a meeting, one time, kindly gave to me.

OK. So what other symmetry restrictions are there? Having shown that inversion will

not permit any third-rank tensor property, we have gone from 32 point groups, lost interest in the 11 centrosymmetric point groups. And so there are only 21 piezoelectric point groups.

And the way we would plod through all 21 of them would be to simply define, starting with a twofold axis. Let's say a twofold axis parallel to x3 would correspond to a change of axes $c_{i,j}$ that describes the new axes in terms of the original ones that consisted of minus 1, 0, 0; 0, minus 1, 0; 0, 0, 1. So if we look at an element in the piezoelectric matrix-- something like $d_{1,6}$-- $d_{1,6}$ actually corresponds in tensor notation to $d_{1,3,2}$ plus $d_{1,2,3}$. And $d_{1,3,2}$ prime is going to be $c_{1,i} c_{3,j} c_{2,k}$ times all of the original tensor elements $d_{i,j,k}$.

The only term of the form c_i, something that is non-zero is $c_{1,1}$. And that has a value, minus 1. The only term of the form c_3, something which is non-zero is $c_{3,3}$. And that turns out to be plus 1. c_2, something, the only form that's non-zero is $c_{2,2}$. And that has value, minus 1.

And this should be times d. And the only value of i that stayed was 1. The only value of j that stayed was 3. And then only value of k that stayed was 2.

So this says that $d_{1,3,2}$ should be equal to $d_{1,3,2}$, which I don't like, unless it's supposed to be negative. I did something wrong here. Well, you see how easy it is, even if it didn't turn out right.

And the form of the tensor for monoclinic crystal of symmetry 2, with a twofold access parallel to x3, has as shown in the lower left-hand corner of the handout on symmetry restrictions, it has eight non-zero terms. If you do the same thing for a mirror plane perpendicular to x3, you find that there are 10 non-zero terms. So we don't have the situation where all of the point groups that are able to show the property within a given crystal systems like monoclinic have exactly the same form of the property tensor.

In fact, you'll notice a curious correspondence between the restrictions for symmetry 2 and symmetry m. All the terms that are 0 in symmetry 2 are non-zero in symmetry

m, and vice versa. All the non-zero terms in symmetry 2 are 0 in symmetry m. And the reason for that is simply that this is the form of the direction cosine scheme for symmetry 2. The form of the direction cosine scheme for symmetry m, where the m is perpendicular to x3, would have the form 1, 0, 0; 0, 1, 0; 0, 0, minus 1.

So ci,j, for a mirror plane perpendicular to x3, is exactly the negative of the direction cosine scheme for a twofold axis parallel to x3. And since the number of direction cosines is odd, this means that everything that has an equality between the di,j,k's for m would have the transformed element be the negative of the original one for 2, and vice versa. So that's why there's this complementary form of the tensors for the two monoclinic symmetries.

Point out a couple of curious things in the tables. You really have to go through 21 of the symmetries independently. And you find that some of them do come out the same. Symmetry 4 bar 3m and symmetry 2:3 have restrictions of the same form. But for the most part-- Yes?

AUDIENCE: I know why this is wrong.

PROFESSOR: Why is that wrong?

AUDIENCE: Because your notation of d1,6 is in fact not d1,3,2 but d1,2,1. Since you have two same indices, [? d3 ?] and minus [? d1. ?]

PROFESSOR: OK. d1,6; d1,6. Ah, of course. Of course. d1,2,6 is the one up here. And that's 1, 2 and 2, 1 for the strains. OK. Thank you.

So this is d1,1,2 plus d1,2,1. And so we would have 1 by 1,k and 2,k. 1, 1, and 2. And 1, 1, 1. 1,i; 1,j; and 2,j for this one. So we'd have c1,1 c1,1 c2,2. c c1,1 is minus 1; c1,1 is minus 1; c2,2 is minus 1.

So d1,1,2 is minus d1,2,1, which means they have to be identically 0. Thank you. I'm sure nobody cares at this point. Very good.

OK. Another curious thing that happens is that for cubic symmetry 4:3:2, it's

acentric. But every single modulus is 0. And the reason, the explanation, is there's so many different transformations which have to leave the tensor invariant that the poor tensor just can't do it. It gives up, packs up, and goes home, leaving all the elements zero. Just no way you can get all the qualities to be satisfied.

Another curious result that I point out for some of the hexagonal symmetries for symmetry 3:2, for symmetry 6:2:2, and for 3 over m and 6 bar 2m, all the elements of the form d3, something are identically 0. Which says you simply cannot create a polarization that has a component P3. You just cannot create a polarization perpendicular to the axis of high symmetry. Yeah. Got a question?

AUDIENCE: So all of the cubic that has 4:3:2 symmetry or [? 4:1:0 ?], that means you can't get--

PROFESSOR: You just don't have any piezoelectric effect, even though the crystal is not centrosymmetric, requiring that all those transformations leave the tensor invariant; simultaneously, require that everything has to be 0.

One final thing, and then I'm running a little bit over. But I'd like to go on to other aspects of piezoelectricity during the next hour. Remember something that we've shown and which I've asked you to look into again on one of the problem sets, and that is that the trace of the strain tensor gives you the change in volume. The first 3 by 3 blocks of this tensor that we've been looking at, were we to write the elements of strain, epsilon i,j, in terms of d-- epsilon j,k-- in terms of di,j,k times e sub k, it is this block in here which enters into the terms epsilon 1,1; epsilon 2,2; and epsilon 3,3.

These terms give you the fractional change in volume. And the elements that are involved in the piezoelectric matrix are these terms in here. So if all of those terms are 0, it turns out that when you apply a stress and look at the electric field, or apply an electric field and look at the strain, if you apply a field and all of those nine terms are 0, you cannot create a strain.

So there's no volume change. There can be a strain, but no volume change. The only deformation you can create is pure shear.

So if you drive a piezoelectric oscillator with an electric field, for those property tensors for which that first 3 by 3 block are all zero, the thing can shear-- in the water, for example-- but it can't pulse. It can't have a volume change. And if you were to create a transducer for sonar applications, what you would want to do is to have piezoelectric device which expanded this way, to create a sound wave going through the water. If it just goes back and forth, it's going to slosh back and forth in the water and not create any sonic wave that could be used in sonar.

Now that is true for elements being identically 0. It turns out it's also true for elements such as 4 bar, where two of those terms in the 3 by 3 block are the negative of one of the other. These will also have no volume change. So that's another very curious property of piezoelectric response that follows from these symmetry restrictions.

OK. That's enough for our first session. When we come back, we'll look at the converse piezoelectric effect. And we'll also look at representation surfaces for specific piezoelectric devices and ask if it's possible for a third-rank tensor to have a representation surface that's analogous to the representation quadric for second-rank tensors. So I'm sure you'll all want to come back and hear the answer to that question.