3.60 Symmetry, Structure and Tensor Properties of Materials

Problem Set No. 16

1. The piezoelectric moduli, d_{ijk}, relate polarization, P_i to the nine elements of stress σ_{jk}:

$$P_i = d_{ijk}\sigma_{jk}, \tag{1}$$

the "direct piezoelectric effect". Since the stress tensor is symmetric, only six components of stress are unique, and it is convenient to write this relation in matrix form (valid only in one particular coordinate system)

$$P_i = d_{ij}\sigma_k \quad \text{where } k = 1 \text{ to } 6. \tag{2}$$

In order to use this simple matrix form it is necessary to lump together two piezoelectric moduli (which are physically inseparable in a measurement of the direct effect) into a single matrix element. For example,

$$d_{123}\sigma_{23}+d_{132}\sigma_{32} = (d_{123}+d_{132})\sigma_{23} \equiv d_{14}\sigma_4. \tag{3}$$

The same piezoelectric moduli may also be used to describe the "converse piezoelectric effect", if the arrangement of tensor elements is suitably interchanged:

$$\varepsilon_{jk} = d_{ijk}E_i \tag{4}$$

(a) Show that relation (4) requires $d_{ijk} = d_{ikj}$.

(b) If we attempt to write this relationship in matrix form,

$$\varepsilon_j = d_{ij}E_i \tag{5}$$

we encounter problems with the bothersome factor of 2 which we absorbed in the definition of matrix elements as in equation (3). By writing any necessary equations of (4) term-by-term, show that we can salvage a matrix relation of the form of (5) only if we agree to the definition

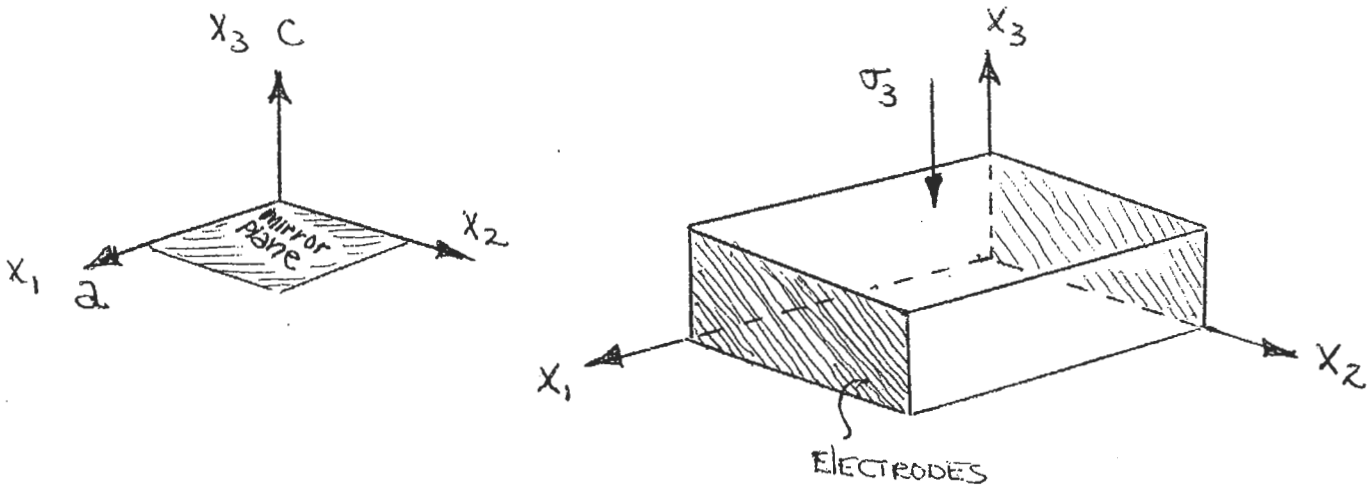
$$\begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \longrightarrow \begin{bmatrix} \varepsilon_1 & \tfrac{1}{2}\varepsilon_6 & \tfrac{1}{2}\varepsilon_5 \\ \tfrac{1}{2}\varepsilon_6 & \varepsilon_2 & \tfrac{1}{2}\varepsilon_4 \\ \tfrac{1}{2}\varepsilon_5 & \tfrac{1}{2}\varepsilon_4 & \varepsilon_3 \end{bmatrix}$$

2. The administration of the Center for Materials Science has become uneasy about certain faculty in Building 13 who are accumulating an inordinate volume of books, notes and ungraded papers in their offices. They fear collapse of the floor and precipitous deposition of said faculty and possessions into the office directly below.

A team of experts has devised a piezoelectric device termed the SOUP cell (Surveillence Of Untidy Professors) to monitor the deflection of the floor. The cell will employ a monoclinic crystal of a new piezoelectric material with symmetry m. When referred to axes such that the mirror plane is normal to x_3 and x_1 is along the $\underline{a}$ axis, the appropriate form of the piezoelectric modulus matrix d_{ij} is

$$\begin{bmatrix} d_{11} & d_{12} & d_{13} & 0 & 0 & d_{16} \\ d_{21} & d_{22} & d_{23} & 0 & 0 & d_{26} \\ 0 & 0 & 0 & d_{34} & d_{35} & 0 \end{bmatrix}$$

The surface of the cell which is normal to x_3 is to be attached to the ceiling. The crystal will be cut in the shape of a rectangle (and disguised as one of our light fixtures to avoid embarrassment) and electrodes attached to a pair of surfaces parallel to x_3 to permit measurement of the induced charge which will be proportional to the stress, σ_3, exerted by the floor. The output of the device will be monitored in the administrative office, thus permitting, if a signal is recorded, spread of the cry of alarm,"The SOUP's on!!".



Your assignment is to determine the orientation, about x_3, in which surfaces should be ground on the crystal for application of the electrodes if the cell is to have optimum sensitivity.

(a) Let us define the SOUP modulus as the proportionality constant between the charge density induced on the surface normal to x_1 and the compressive stress σ_3. Evaluate the SOUP modulus for the setting of axes and the form of the tensor given above.

(b) Obtain an expression for the SOUP modulus as a function of the orientation of the surfaces bearing the electrodes. (The surfaces are always parallel to x_3--that is, the compressive stress is always along x_3 and the crystal is always oriented with its c axis along this direction.) Determine the orientation which would provide a maximum value for the modulus.

(c) A member of the above-mentioned team of experts has a bright idea: If the output of the cell signals the danger of imminent collapse of a floor, let a large voltage immediately be applied across the electrodes to create a strain ε_3 which will help support the floor. For the orientation of this crystal in (a) will this action actually produce a component of strain ε_3? Will it do so, as well, for the orientation of the electrodes which you recommend in (b)?