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## 3.22 Mechanical Properties of Materials Spring 2008

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## Problem Set #3

Due: Friday, March 21 by 5:00 PM

1. Because metallic glasses have an amorphous atomic structure, many researchers believe they cannot “strain harden” like their crystalline counterparts (a topic we will cover in depth later). At present, you can understand this to mean that the stress required to continue deformation of the material to strains greater than the yield strain  $\epsilon(\sigma_y)$  increases monotonically, often modeled as a power-law of the form  $\sigma = K\epsilon_p^n$  where  $\epsilon_p$  is the plastic strain,  $n$  is the material strain hardening exponent, and  $K$  is a material constant related to  $\sigma_y$ . In other words, after yielding the material becomes “harder (i.e. more difficult) to strain.” Regardless of this assumption, you decide to have a UROP in your group conduct some experiments to consider whether strain hardening does occur in a particular metallic glass you have synthesized. You have the UROP compress a cylindrical sample of bulk metallic glass that is 6 mm long and 3 mm in diameter to failure. After the tests, the UROP excitedly presents the data to you (which can be found in 3-1.xls), claiming to have proven with these data that strain hardening in metallic glasses is possible! As she hyperventilates talking about the *Nature* article she wants to write, you realize that she is wrong (most likely because she did not have the opportunity to yet take 3.22).
  - (a) Explain why she is wrong. Do this by calculating and discussing the  $\sigma - \epsilon$  response(s) and the strain hardening exponent  $n$  for the material. Compare your results with what is expected of a material that does strain harden.
  - (b) Determine the elastic and plastic strain at failure.
  - (c) Not one to give up easily, you think that you may be able to get the metallic glass to strain harden by applying a cyclic load (with the minimum load being 0 N) through the plastic region. Show what the resulting stress-strain curve would look like if you applied three cycles before failure.
2. You are given a material and told that it is elastically isotropic and exhibits a yield strength  $\sigma_y$  of 950 MPa. The material is being used in an application where it experiences a stress state of

$$\sigma_{ij} = \begin{vmatrix} 0 & 0 & 300 \\ 0 & -400 & 0 \\ 300 & 0 & -800 \end{vmatrix} \text{ MPa}$$

- (a) According to the von Mises and Tresca criteria, respectively, does the material yield?
  - (b) Plot the yield surfaces (on the same graph) of the material based on the von Mises and Tresca criteria.
  - (c) You decide to examine the material more closely by performing your own uniaxial tension and compression tests. The stress-strain data for these tests are found in 3-2.xls. Plot these data on the same graph as you made in (b). Discuss how the von Mises and Tresca criteria predict yielding in the material. Is there a better criterion to use for this material? Explain.
3. Noting that the von Mises and Tresca criteria deviate for specific stress states in the yield surface considered above, determine the following:

- (a) The stress tensor  $\sigma_{ij}$  corresponding to the stress state(s) at which there is maximal difference between these two yield criteria, expressed as a matrix and as a representative volume element (RVE) of the material. This will correspond to the biggest gap(s) between these two predictions in the graphical yield surface.
  - (b) The magnitude of this difference, expressed first in terms of an algebraic expression including the  $\sigma_y$  of the material and second in terms of a percentage basis [%].
  - (c) *Which* of these two criteria more accurately predicts the stress states corresponding to material yielding in pure metals and metallic alloys, and *why* does this tend to be the more accurate predictor? To answer this, you will need to rely on and show/cite experimental data for selected materials.
4. The yield strength  $\sigma_y$  of many materials is poorly described by von Mises and Tresca; these include porous materials, glassy polymers, granular media, and biological materials. For such materials,  $\sigma_y$  depends in part on the state of hydrostatic stress  $\sigma_{ii}$  and thus the first invariant of  $\sigma_{ij}$ ,  $I_1$ . Here, you will explore some of these pressure-sensitive yield criteria.
- (a) Mohr and Coulomb discussed continuum mechanics and particle interactions, respectively. Their ideas enabled modification of the yield criteria to reflect the fact that  $\sigma_y$  may differ in uniaxial tension vs. compression. This criterion is commonly used for bulk metallic glass and soils, which fail at lower applied stresses in tension than in compression. It linearly relates the effect of superposed normal and shear stresses on the stress state required to yield the material, and Mohr's circle helps us easily understand this. Assume an imposed principal stress state in a material plane; Mohr-Coulomb expresses the yield criteria in terms of the stress state at maximal shear stress resulting from  $\sigma_1$  and  $\sigma_3$ , and can be expressed as the material yielding when:

$$\tau = \langle \sigma \rangle \sin \alpha \quad (1)$$

where  $\langle \sigma \rangle$  is the average normal stress corresponding to  $\tau$ , and  $(\tau, \langle \sigma \rangle)$  is the coordinate on Mohr's circle defined at the point of max shear stress for defined  $\sigma_1 > 0$  and  $\sigma_3 > 0$ , and  $\alpha$  is the friction angle. Physically,  $\alpha$  is the angle that would naturally arise if you poured the granular material onto a surface due to friction between the particles, and is usually between  $15^\circ$  and  $50^\circ$ . In terms of Mohr's circle, it is the angle between the vertical line connecting  $(\tau, \langle \sigma \rangle)$  to the  $\tau = 0$  horizontal axis, and the tangent of the circle that intersects the point  $(\tau = 0, \sigma = 0)$ . Draw this Mohr's circle, and restate the Mohr-Coulomb yield criterion only in terms of these normal principal stresses and the material friction angle.

- (b) Drucker and Prager developed the creatively named Drucker-Prager yield criterion originally for soil mechanics and it is often applied to composite materials such as cement or concrete. It accounts for normal and shear stresses, and can be stated as:

$$f = \sqrt{J_2} - \alpha I_1 - k \quad (2)$$

where  $J_2$  is the second invariant of the deviatoric stress tensor  $s_{ij}$  and  $\alpha$  and  $k$  are material properties that reflect the friction angle and cohesion, respectively. When  $f$  reaches a critical value corresponding to yielding under a uniaxial stress state, the material will

- yield. Express  $f$  in terms of the stresses  $\sigma_{ij}$  that define  $I_1$  and  $J_2$ , and determine the value of  $f(\alpha, k)$  that corresponds to yielding under an applied uniaxial stress state.
- (c) Mohr-Coulomb and Drucker-Prager are similar in relation to each other as Tresca and von Mises. Both account for superposed normal and shear stresses, but one is more conservative than the other. Graphically represent the M-C and D-P yield locii on the same  $\sigma_1/\sigma_y$  vs.  $\sigma_2/\sigma_y$  graph, and state *why* one is more conservative than the other in predicting yielding of pressure-sensitive materials.
5. Material stiffness and strength are very different properties attributed to very different atomistic mechanisms. In 60 words or less (think elevator conversation), clearly and accurately state the difference between these two concepts in a way that should be perfectly clear to any engineer, physicist, or chemist.
6. We have discussed that dislocations move in response to shear stresses less than the theoretical shear strength of a crystal, and that the motion of each dislocation contributes unit slip of magnitude  $|\mathbf{b}|$ .
- (a) Express the plastic shear strain  $\gamma^p$  that would result from a single edge dislocation gliding along its slip plane all the way to the free surface. Here, consider a rectangular single crystal block of height  $h$ , width across the page  $w$  and depth into the page  $L$ ; first express  $\gamma^p(b, h)$  and then express  $\gamma^p(A, V)$  where  $A$  is the glide plane area and  $V$  is the volume of the block.
- (b) Now consider a single edge dislocation in the center of the perfect rectangular-prism box, gliding by an incremental slip inside the block  $dx$ . Express the incremental shear strain  $d\gamma^p$  in terms of  $b, L, dx$  and  $V$ .
- (c) Now consider the case of  $n$  dislocations, and express this incremental shear strain in terms of dislocation density  $\rho$  (which is always defined as total dislocation line length per unit volume of material),  $b$  and the incremental slip  $dx$ .
- (d) Finally, express the time derivative of  $d\gamma^p$  in terms of the time derivative of displacement  $dx/dt$ . You will have derived the Orowan equation, which predicts the plastic shear strain rate possible in a material of known dislocation density, Burgers vector, and dislocation velocity  $\mathbf{v}$ .
- (e) Assume dislocation velocity is equivalent to the speed of sound in Cu, and determine the plastic shear strain rate magnitude for annealed, single crystal Cu.
7. Complete PS3 wiki questions with your Special Topic colleagues.