3.205 Problem 7 Solutions

Problem 1

$$c(r,t) = \frac{nd}{(4\pi Dt)^{\frac{3}{2}}} \exp\left(\frac{-r^2}{4Dt}\right)$$

a)

$$\vec{J}(r,t) = -D\nabla c$$
$$\nabla c = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\begin{aligned}
\frac{\partial c}{\partial r} &= \frac{nd}{(4\pi Dt)^{\frac{3}{2}}} \times \frac{-2r}{4Dt} \exp\left(\frac{-r^2}{4Dt}\right) \\
&= \frac{-ndr}{(16\pi^{\frac{3}{2}} D^{\frac{5}{2}} t^{\frac{5}{2}})} \exp\left(\frac{-r^2}{4Dt}\right) \\
&= \frac{-ndr}{(16\pi^{\frac{3}{2}} D^{\frac{5}{2}} t^{\frac{5}{2}})} \exp\left(\frac{-r^2}{4Dt}\right)
\end{aligned}$$

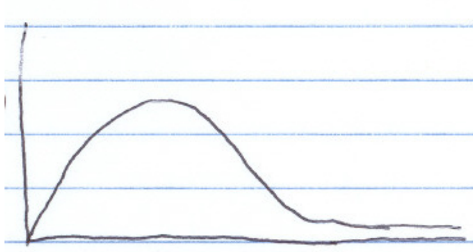
$$\vec{J}(r,t) = \frac{ndr}{(16\pi^{\frac{3}{2}} D^{\frac{3}{2}} t^{\frac{5}{2}})} \exp\left(\frac{-r^2}{4Dt}\right)$$

b) Rate of accumulation of species i

$$\begin{aligned}
\frac{\partial c_i}{\partial t} &= \frac{nd}{(4\pi D)^{\frac{3}{2}}} \cdot \left(\frac{3}{2}\right) t^{\frac{-5}{2}} \exp\left(\frac{-r^2}{4Dt}\right) + \frac{nd}{(4\pi Dt)^{\frac{3}{2}}} \left(\frac{-r^2}{4Dt^2} \cdot (-1)\right) \exp\left(\frac{-r^2}{4Dt}\right) \\
&= \frac{-3nd}{16(\pi D)^{\frac{3}{2}} t^{\frac{5}{2}}} \exp\left(\frac{-r^2}{4Dt}\right) + \frac{ndr^2}{(2(\pi D)^{\frac{3}{2}} t^{\frac{7}{2}} \cdot D)} \exp\left(\frac{-r^2}{4Dt}\right) \\
&= \frac{nd}{16(\pi D)^{\frac{3}{2}} t^{\frac{5}{2}}} \left[-3 + \frac{r^2}{2Dt}\right] \exp\left(\frac{-r^2}{4Dt}\right)
\end{aligned}$$

c)

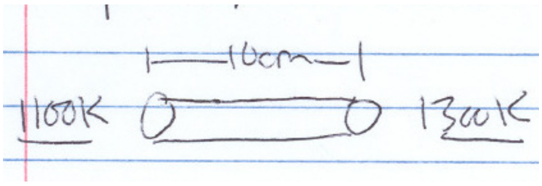
c(r, t)



time

Increases to a maximum, then decreases monotonically

Problem 2



a)

$$\begin{aligned} J_C &= L_{CC}X_C + L_{CQ}X_Q \\ J_Q &= L_{QQ}X_Q + L_{QC}X_C \end{aligned}$$

$$L_{CQ} = L_{QC}$$

$$X_C = \frac{-d}{dx} \qquad X_Q = \frac{-1}{T}\frac{dT}{dx} \tag{1}$$

$$\tag{2}$$

b)

$$J_C = L_{CC}X_C + L_{CQ}X_Q$$
$$X_C \to 0$$
$$J_C \to 0$$
$$0 = L_{CQ}X_Q$$
$$X_Q = \frac{-1}{T} \neq 0 \text{ there } \textit{is} \text{ a temperature gradient.}$$

For carbon-atom concentration to remain uniform $L_{CQ} = 0$.

c)

$$J_C = L_{CC}X_C + L_{CQ}X_Q$$
$$X_C \to 0$$
$$X_Q = \frac{-1}{T}\nabla T = \frac{-1}{T}\frac{dT}{dx}$$
$$J_C = L_{QC}\left(\frac{-1}{T}\right)\frac{dT}{dx}$$
$$J_C > 0, \left(\frac{-1}{T}\right) < 0$$

For J_C to end hotter end of bar, $L_{QC} < 0$.