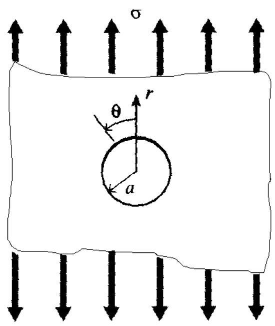
Stresses Around a Circular Hole

Uniaxial tension, hole of radius a:



Boundary conditions:

$$\left.\begin{aligned} \sigma_r &= \frac{\sigma}{2}\left(1+\cos 2\theta\right) \\ \sigma_\theta &= \frac{\sigma}{2}\left(1-\cos 2\theta\right) \\ \tau_{r\theta} &= \frac{\sigma}{2}\sin 2\theta \end{aligned}\right\}, r \to \infty$$

$$\sigma_r = \tau_{r\theta} = 0, r = 0$$

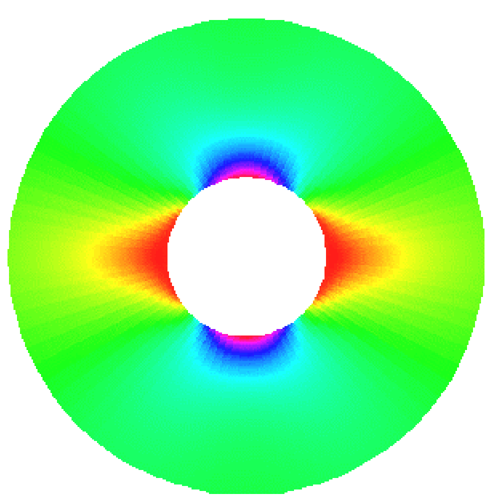
Assumed Airy stress function: $\phi = f(r)\cos 2\theta$

$$\nabla^4 \phi(r,\theta) = 0 \to f(r) = Ar^2 + Br^4 + C\frac{1}{r^2} + D$$

Evaluate constants from boundary conditions, then

$$\sigma_\theta = \frac{\partial^2 \phi}{\partial r^2} = \frac{\sigma}{2}\left(1+\frac{a^2}{r^2}\right) - \frac{\sigma}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_{\theta,\max} = 3\sigma \ @ \ \ r = a, \theta = \pi/2$$



3.11 Mechanics of Materials
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