Materials Selection for Mechanical Design II

A Brief Overview of a Systematic Methodology

Material <u>and</u> Shape Selection

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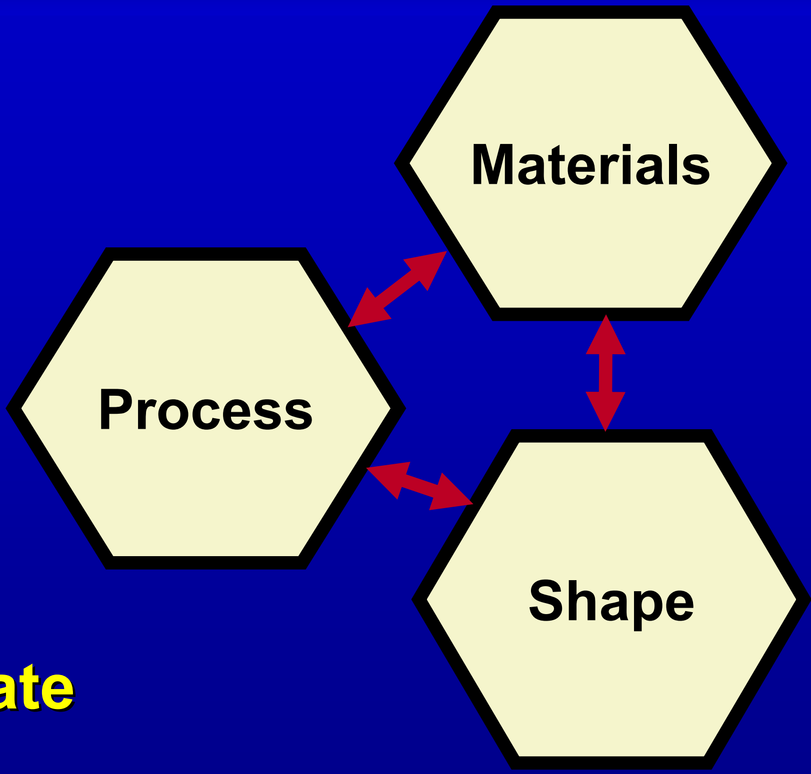
Method for Early Technology Screening

- Design performance is determined by the combination of:
  - Shape
  - Materials
  - Process
- Underlying principles of selection are unchanged
  - BUT, do not underestimate impact of shape or the limitation of process



Material and Shape Selection

- Performance isn't just about materials - shape can also play an important role
- Shape can be optimized to maximize performance for a given loading condition
- Simple cross-sectional geometries are not always optimal
  - Efficient Shapes like I-beams, tubes can be better
- Shape is limited by material
  - Wood can be formed only so thin
- Goal is to optimize both shape and material for a given loading condition

Loading Conditions and Shape

- Different loading conditions are enhanced by maximizing different geometric properties
- Area for tension
- Second moment for compression and bending
- Polar moment for torsion

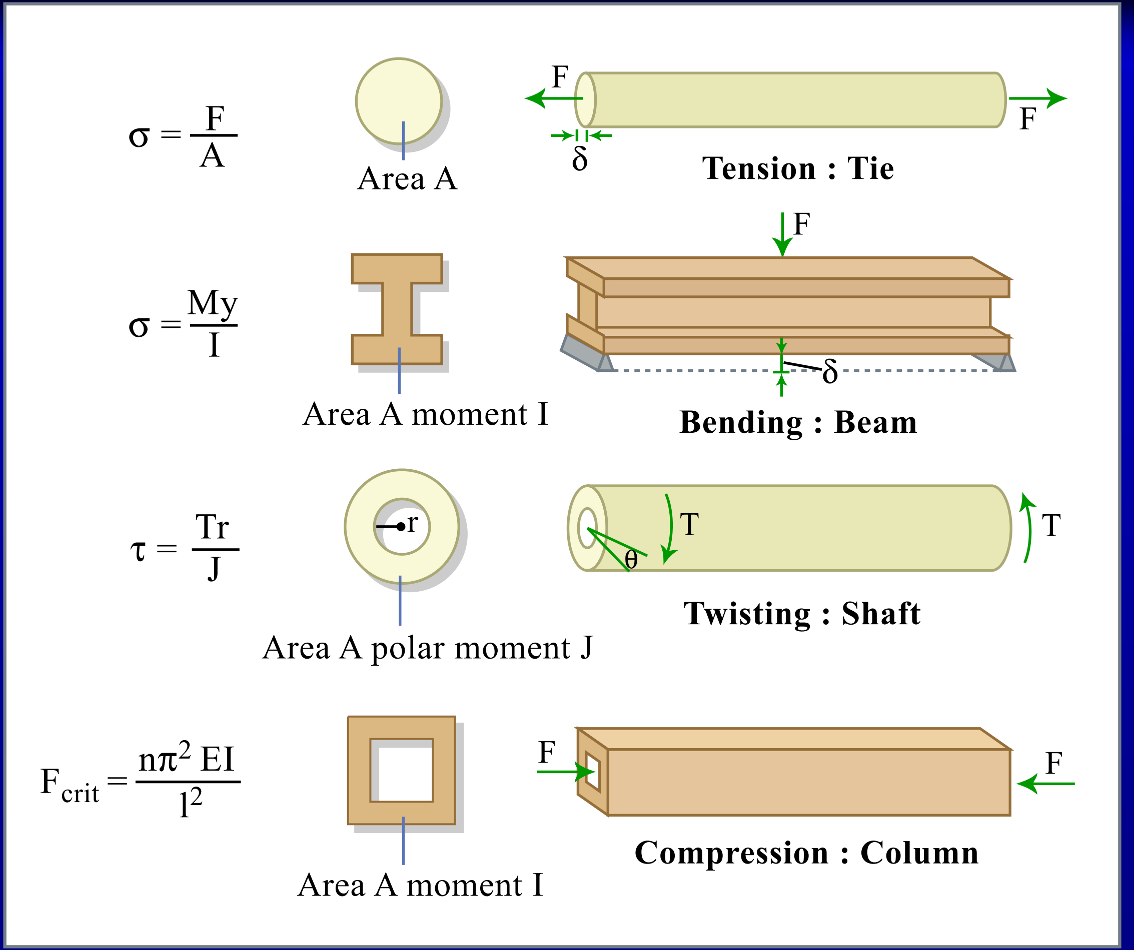
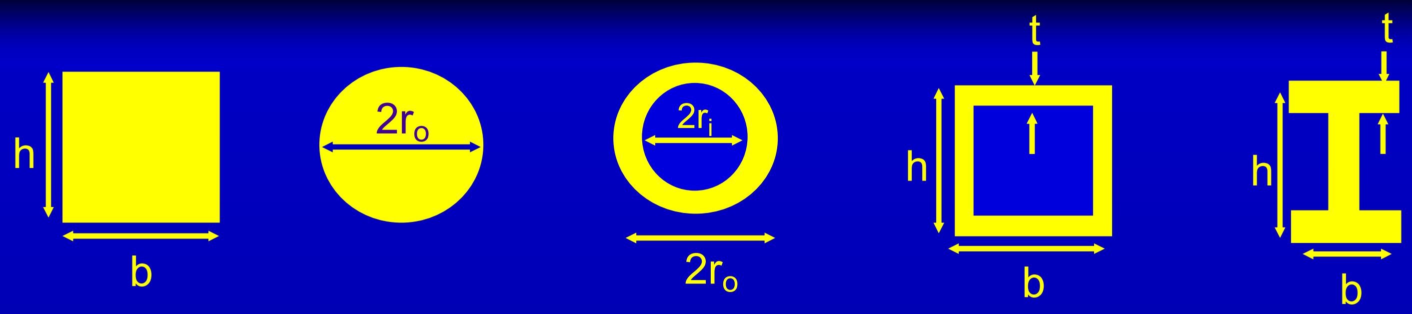


Figure by MIT OCW.

Shapes and Moments



Area	bh	πr^2	$\pi\left(r_o^2 - r_i^2\right)$	$2t\left(h+b\right)$	$2t\left(h+b\right)$
Second Moment	$\frac{bh^3}{12}$	$\frac{\pi}{4}r^4$	$\frac{\pi}{4}\left(r_o^4 - r_i^4\right)$	$\frac{1}{6}h^3 t\left(1+3\frac{b}{h}\right)$	$\frac{1}{6}h^3 t\left(1+3\frac{b}{h}\right)$
Polar Moment	$\frac{bh^3}{3}\left(1-0.58\frac{b}{h}\right)$	$\frac{\pi}{2}r^4$	$\frac{\pi}{2}\left(r_o^4 - r_i^4\right)$	$\frac{2tb^2h^2}{\left(h+b\right)}\left(1-\frac{t}{h}\right)^4$	$\frac{2}{3}bt^3\left(1+4\frac{h}{b}\right)$

Shape Factor Definition

- Shape factor measures efficiency for a mode of loading given an equivalent cross-section
  - "Efficiency": For a given loading condition, section uses as little material as possible
- Defined as 1 for a solid cross-section
  - Higher number is better, more efficient

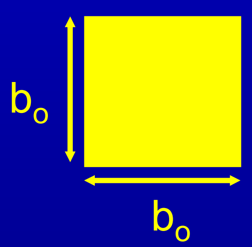
$$\phi^e = \frac{S}{S_o}$$

For elastic cases:
ϕ = shape factor
S = stiffness of cross-section under question
S_o = stiffness of reference solid cross-section

Shape Factor for Elastic Bending

$$\phi_B^e = \frac{S}{S_o} = \frac{EI}{EI_o} = \frac{I}{I_o}$$

Reference solid cross-section



$$I_o = \frac{b_o^{\ 4}}{12} = \frac{A_o^{\ 2}}{12}$$

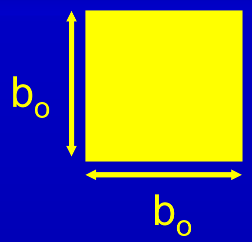
Compare sections of same area ⇒

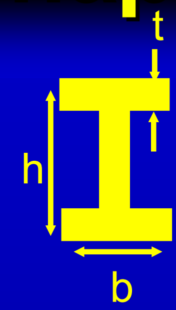
$$A_o = A$$

$$\phi_B^e = \frac{I}{I_o} = \frac{12I}{A^2}$$

Notice that shape factor is dimensionless

I-Beam Elastic Bending Shape Factor





t = 0.125
h = 3
b = 1

$$A_o = b_o^2$$

$$b_o = 1$$

$$A_o = 1$$

$$A = 2t\left(h + b\right)$$

$$A = 1 = A_o$$

$$I = \frac{1}{6}h^3t\left(1 + 3\frac{b}{h}\right) = 1.125$$

$$\phi_B^e = \frac{12I}{A^2} = 13.5$$

For these dimensions, the shape increased stiffness *over 13 times* while using the same amount of material!

Is this design possible in all materials?

Materials Limit Best Achievable Shape Factor

- Shape efficiency dependent on material
- Constraints: manufacturing, material properties, local buckling
  - For example, can't have thin sections of wood
- Values in table determined empirically
- Note: previous design not possible in polymers, wood (ϕ^e_B)=13.5

Material	Bending $\left(\phi_B^e\right)_{\max}$	Torsion $\left(\phi_T^e\right)_{\max}$
Structural Steels	65	25
Aluminum Alloys	44	31
GFRP and CFRP	39	26
Polymers	12	8
Woods	6	1

Shape Factors and Material Indices Example: Bending Beam

Mass: $m = AL\rho$

Bending Stiffenss: $S = \frac{F}{\delta} \geq \frac{CEI}{L^3}$

Shape Factor: $\phi_B^e = \frac{I}{I_o} = \frac{12I}{A^2}$

Replace I **in Stiffness using** ϕ_B^e**:** $S = \frac{C}{12}\frac{E}{L^3}\phi_B^e A^2$

Eliminate A **from mass using stiffness:** $m = \left(\frac{12S}{C}\right)^{1/2} L^{5/2}\left[\frac{\rho}{\left(\phi_B^e E\right)^{1/2}}\right]$

Material Index: $M = \frac{\left(\phi_B^e E\right)^{1/2}}{\rho}$

Previously: $M = \frac{E^{1/2}}{\rho}$

Shape Factors and Material Indices: Beams

Objective: **Minimize Mass**

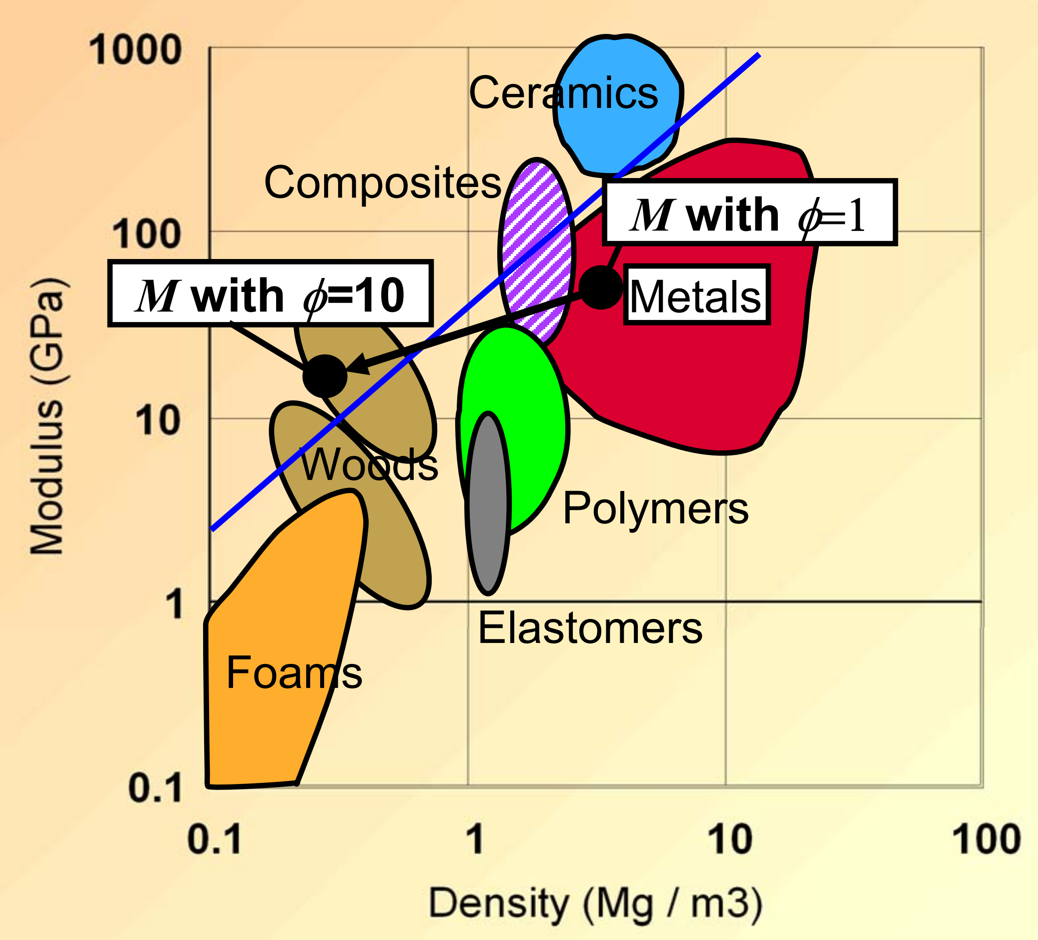
Performance Metric: **Mass**

Loading	Stiffness Limited	Strength Limited
Tension	E/ρ	σ_f/ρ
Bending	$(\phi^e_B E)^{1/2}/\rho$	$(\phi^f_B \sigma_f)^{2/3}/\rho$
Torsion	$(\phi^e_T G)^{1/2}/\rho$	$(\phi^f_T \sigma_f)^{2/3}/\rho$

↖ ***Maximize!*** ↗

Shape Factors Affect Material Choice

- Shape factors can dramatically improve performance for a given loading condition
- The optimal combination of shape and material leads to the best design



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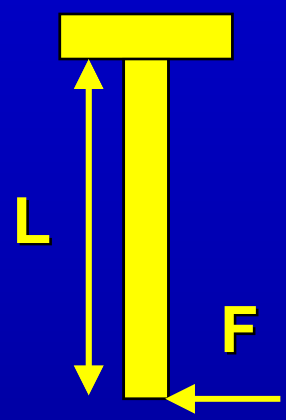
Example Problem: Bicycle Forks

Photos of bicycle forks removed for copyright reasons.

- Bicycle forks need to be lightweight
- Primary constraint can be stiffness or strength
- Toughness and cost can be other constraints

Bicycle Forks: Problem Definition

- **Function:**
  - Forks - support bending loads
- **Objective:**
  - Minimize mass
- **Constraints:**
  - Length specified
  - Must not fail (strength constraint)
- **Free variables:**
  - Material
  - Area: Tube radius OR thickness OR shape



Objective: $m = AL\rho$

Constraint: $\sigma = \dfrac{My_m}{I} = \dfrac{FLy_m}{I} \leq \sigma_f$

Free Variables:

Solid Tube: $A = \pi r^2 \quad I = \dfrac{\pi r^4}{4}$

Hollow Tube: $A \approx 2\pi r t \quad I \approx \pi r^3 t$

Shape: $\phi_B^f = \dfrac{4\sqrt{\pi}Z}{A^{3/2}} \quad Z = \dfrac{I}{y_m}$

Material Indices: Shape specified
Free variable definition important

Solid Section

Free Variable: Area

$$\sigma = \frac{My_m}{I} \leq \sigma_f$$

$$\sigma_f \geq \frac{4FL}{\pi r^3}$$

Solve for r:

$$r = \left(\frac{4FL}{\pi \sigma_f}\right)^{1/3}$$

Substitute into m:

$$m = \pi^{1/3} \left(4F\right)^{2/3} L^{2/3} \left[\frac{\rho}{\sigma_f^{2/3}}\right]$$

Maximize: $M = \left[\frac{\sigma_f^{2/3}}{\rho}\right]$

Hollow Section

Free Variable: Radius

$$\sigma = \frac{My_m}{I} \leq \sigma_f$$

$$\sigma_f \geq \frac{FL}{\pi r^2 t}$$

Solve for r:

$$r = \left(\frac{FL}{\pi t \sigma_f}\right)^{1/2}$$

Substitute into m:

$$m = \left(4\pi F\right)^{1/2} \left(L^3 t\right)^{1/2} \left[\frac{\rho}{\sigma_f^{1/2}}\right]$$

Maximize: $M = \left[\frac{\sigma_f^{1/2}}{\rho}\right]$

Hollow Section

Free Variable: Thickness

$$\sigma = \frac{My_m}{I} \leq \sigma_f$$

$$\sigma_f \geq \frac{FL}{\pi r^2 t}$$

Solve for t:

$$t = \frac{FL}{\pi r^2 \sigma_f}$$

Substitute into m:

$$m = 2F \frac{L^2}{r} \left[\frac{\rho}{\sigma_f}\right]$$

Maximize: $M = \left[\frac{\sigma_f}{\rho}\right]$

Material Index with Shape Free

$$\sigma_f \geq \frac{FLy_m}{I} = \frac{FL}{Z} = \frac{FL4\sqrt{\pi}}{\phi_B^f A^{3/2}}$$

Solve for A:

$$A = \left(\frac{FL4\sqrt{\pi}}{\phi_B^f \sigma_f}\right)^{2/3}$$

Substitute into m:

$$m = \left(4\sqrt{\pi}F\right)^{2/3} L^{5/3} \left[\frac{\rho}{\left(\phi_B^f \sigma_f\right)^{2/3}}\right]$$

Maximize: $M = \left[\dfrac{\left(\phi_B^f \sigma_f\right)^{2/3}}{\rho}\right]$

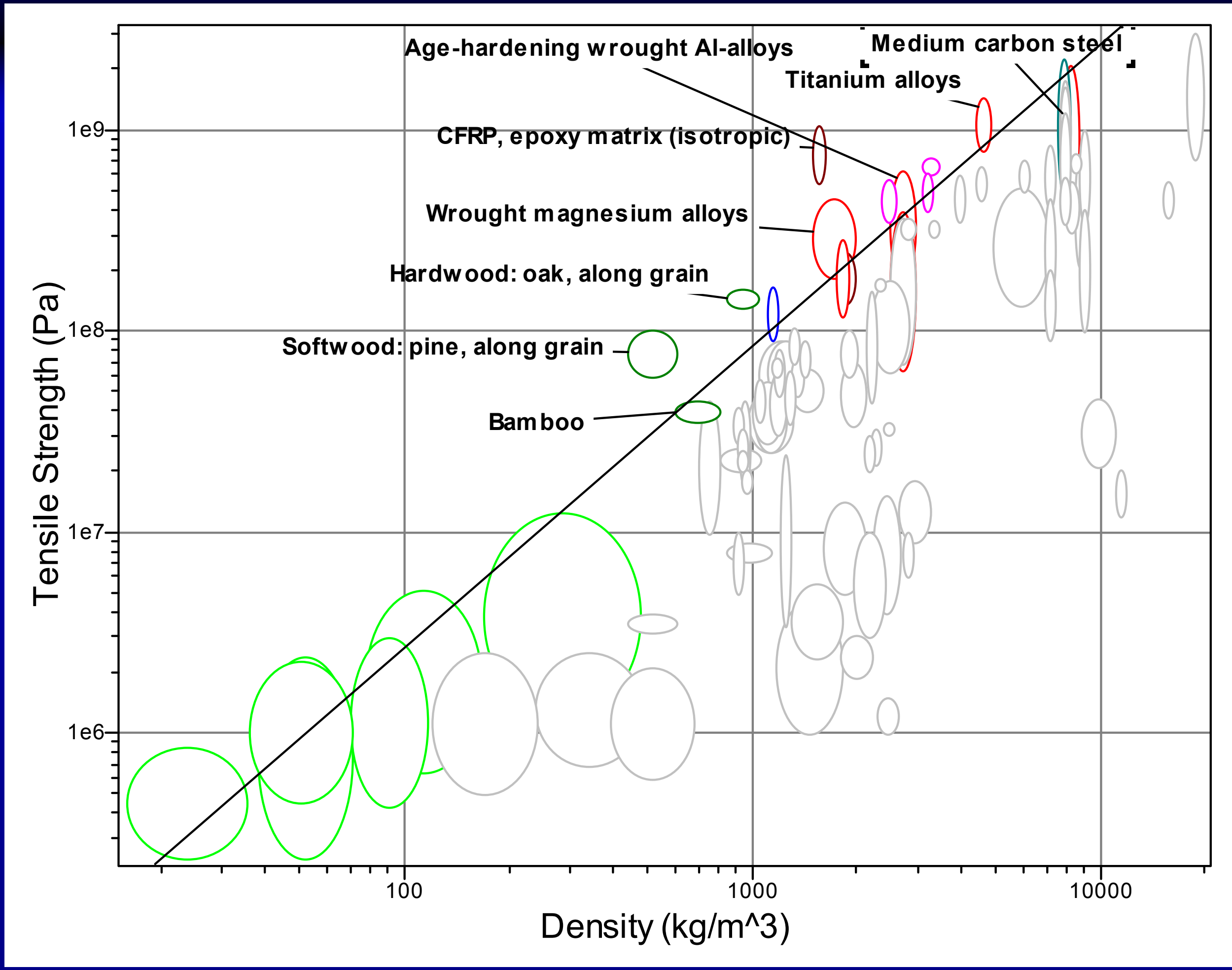
Material indices with shape factors change material selection

Material	σ_f (MPa)	ρ (Mg/m3)	ϕ^f_B	$\sigma_f^{2/3}/\rho$ *	$(\phi^f_B \sigma_f)^{2/3}/\rho$ **
Spruce (Norwegian)	80	0.51	1	36	36
Bamboo	120	0.7	2.2	35	59
Steel (Reynolds 531)	880	7.82	7.5	12	45
Alu (6061-T6)	250	2.7	5.9	15	48
Titanium 6-4	955	4.42	5.9	22	72
Magnesium AZ 61	165	1.8	4.25	17	44
CFRP	375	1.5	4.25	35	91

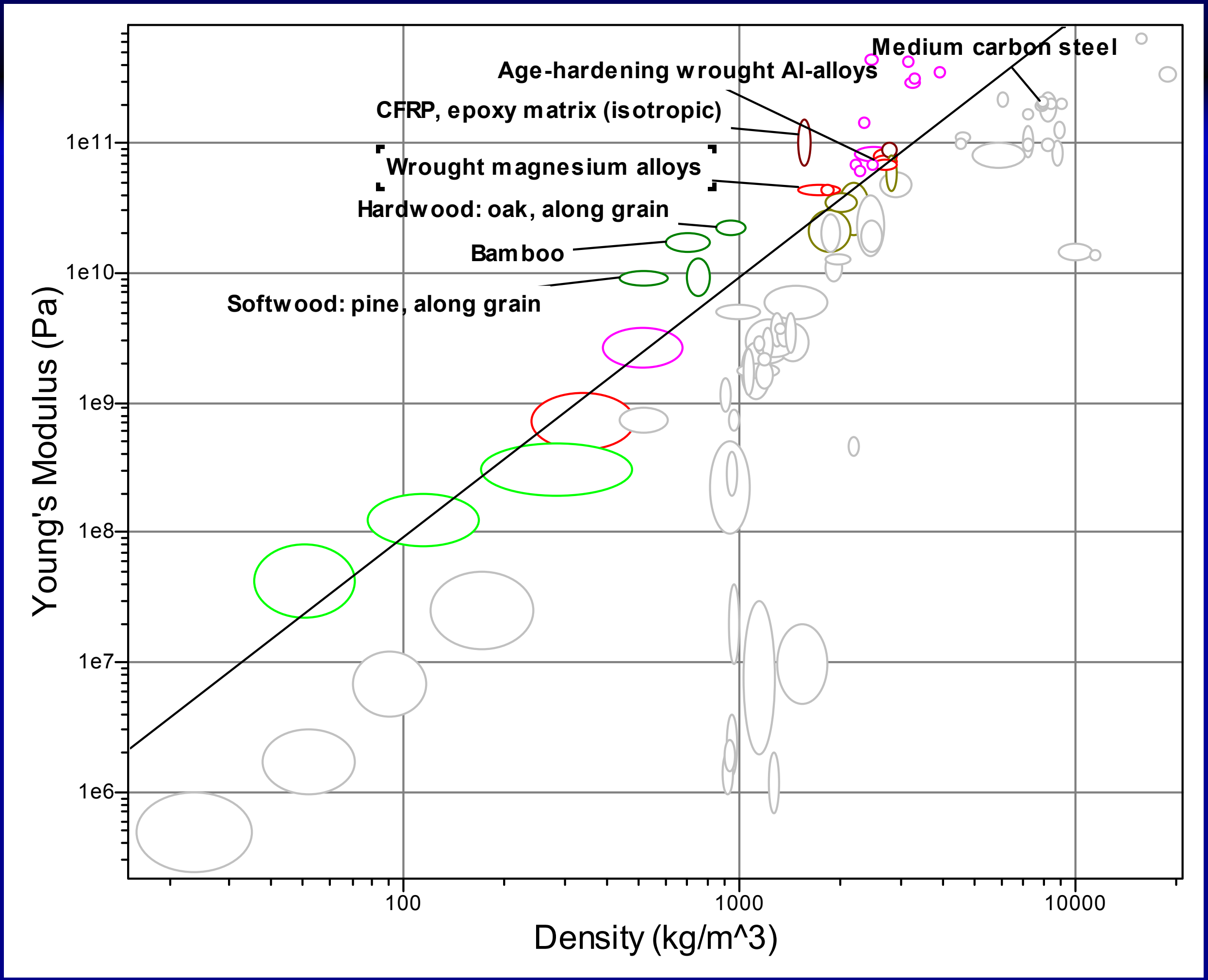
*Material Index w/out shape factor

**Material Index *with* shape factor

Strength Constraint



Stiffness Constraint



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Example of Material Selection including Shape: Floor Joists

Material for floor joists	Wood beam	Steel I-beam
Density (g/cm^3)	~0.58	~7.9
Modulus (GPa)	~10	~210
Material Cost ($/kg)	~$0.90	~$0.65
ϕ^e_B	2.0-2.2	15-25
* $E^{1/2}/C_m\rho$	**~6.1**	~2.8
** $(\phi^e_B E)^{1/2}/C_m\rho$	~8.8	**~12.6**

***Material Index w/out shape factor**

****Material Index *with* shape factor**