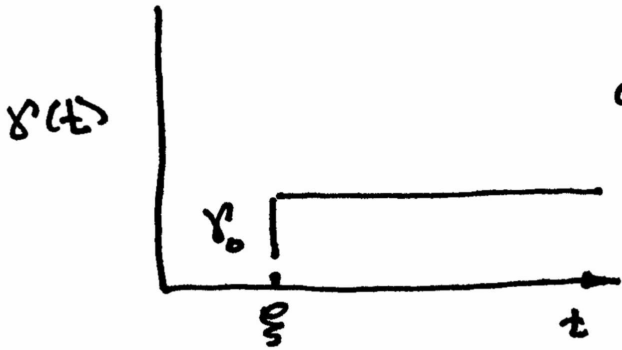
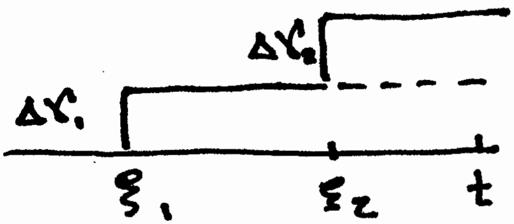
Superposition



$$\sigma(t) = G(t-\xi)\cdot\gamma_0$$



$$\sigma(t) = G(t-\xi_1)\Delta\gamma_1 + G(t-\xi_2)\Delta\gamma_2 + \cdots$$

$$\sigma(t) = \sum_j G(t-\xi_j)\Delta\gamma_j \rightarrow \int_{-\infty}^{t} G(t-\xi)\,d\gamma$$

$$\sigma(t) = \int_{-\infty}^{t} G(t-\xi)\,\frac{d\gamma(\xi)}{d\xi}\,d\xi$$

Derive complex modulus for Zener solid using superposition

Relaxation modulus

```
> G:=t -> G_R+(G_U - G_R)*exp(-t/tau);
```

$$G := t \to G\_R + (G\_U - G\_R)\,\mathbf{e}^{\left(-\frac{t}{\tau}\right)}$$

Sinusoidal strain input (unit magnitude)

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> unprotect(gamma):gamma:=t -> sin(omega*t);
```

$$\gamma := t \to \sin(\omega\, t)$$

Superposition integral for stress output

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> G_star:=int(G(t-xi)*diff(gamma(xi),xi),xi=0..t);
```

Simplify a bit

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> collect(factor(G_star),sin(omega*t));
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$$-\frac{(-\omega^2\, G\_U\, \tau^2 - G\_R)\sin(t\,\omega)}{1+\omega^2\,\tau^2}$$

$$-\frac{\omega\, G\_U\,\tau\,\mathbf{e}^{\left(-\frac{t}{\tau}\right)} - \omega\, G\_U\,\tau\cos(t\,\omega) + \omega\, G\_R\,\tau\cos(t\,\omega) - \omega\,\tau\,\mathbf{e}^{\left(-\frac{t}{\tau}\right)}\, G\_R}{1+\omega^2\,\tau^2}$$