Sandwich structures in nature

- previously, saw sandwich structures efficient in resisting bending, buckling
- sandwich panels also appear in nature:
  - leaves of monocotyledon plants (grasses, corn, iris)
  - skulls (esp. birds)
  - shells of some arthropods (eg. horseshoe crab)
  - cuttlefish bone (mollusk)

Leaves

- leaves must provide for structural support as well as large surface area for photosynthesis
- iris, cattail, ryegrass, giant feather grass - leaves all sandwich str.

Iris leaves

- nearly fully dense ribs (sclerenchyma) running along length of outer surfaces
- ribs separated by a core of foam-like parenchyma cells

Leaves

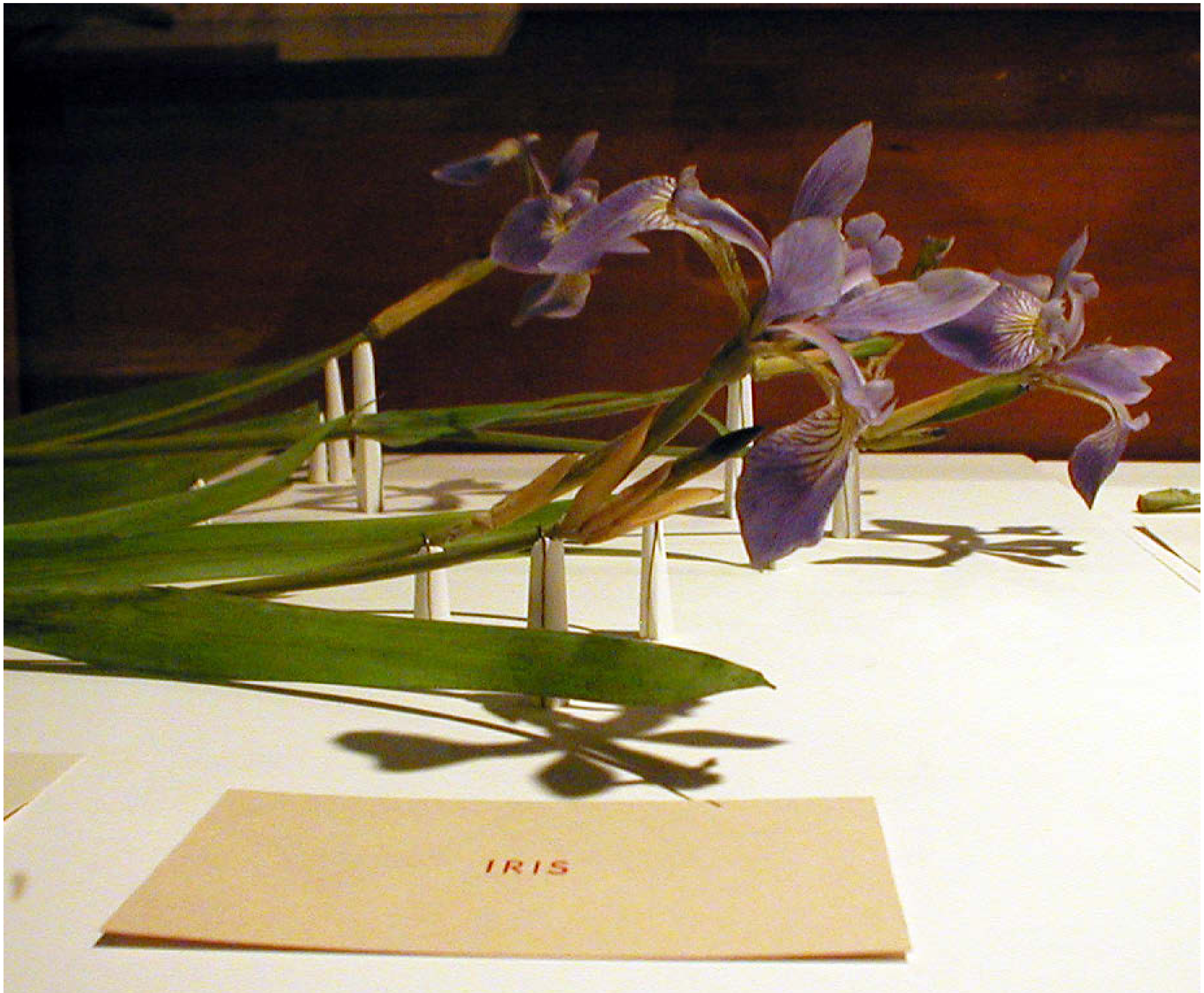
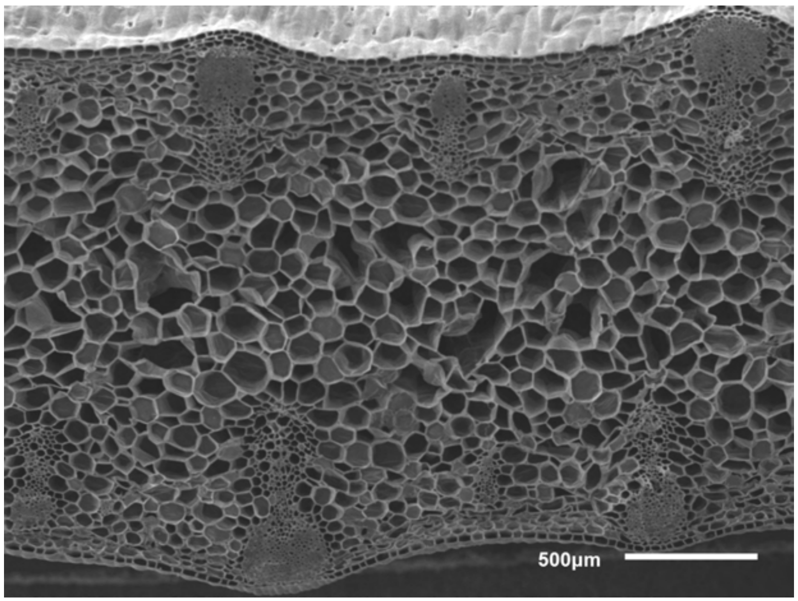
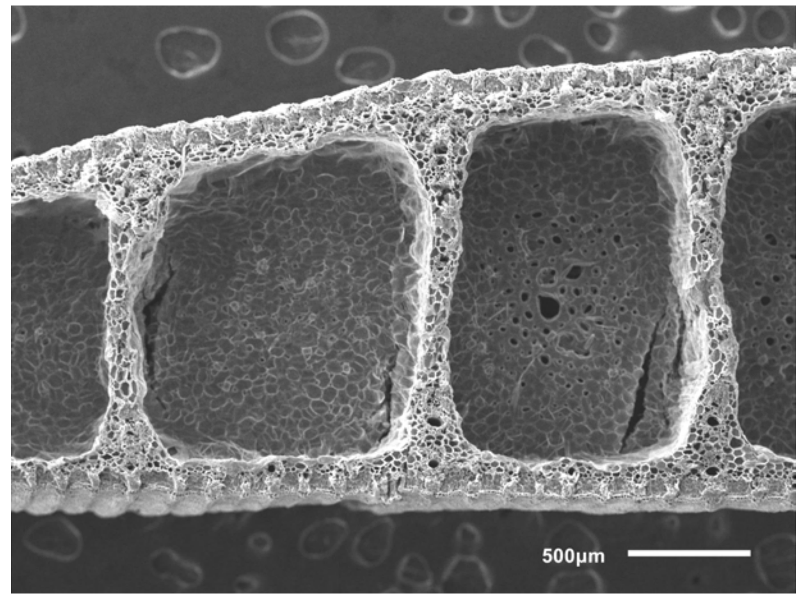


Photo of Blaschka glass flowers (iris) at the Harvard Museum of Natural History. Courtesy of Andrew Kuchling on Flickr.

Leaves



Iris



Cattail
(Bulrush)

Leaves

Iris leaf

- outer face - ribs connected by single layer of roughly square cells
  - jointly act as fibre reinforced composite
- measurements of leaf microstructure summarized in Table
- can analyze leaf as a sandwich structure
- compare analysis with bending tests on fresh iris leaves
  - cantilevers with weights hung from free end ($B_1 = 3$, $B_2 = 1$)

$$\left(\frac{\delta}{P}\right)_{calc} = \frac{2\,l^3}{3\,E_f\,b\,t\,c^2} + \frac{l}{G_c^*\,b\,c}$$

t, c measured from micrographs (Table)

b, l from beam bending tests

E_f, G_c^* need to estimate

E_f - can be estimated from $E_{rib} V_{rib}$ in face (neglect contrib of square cells in face).

- ribs - sclerenchyma
- previous studies - sclerenchyma from grass leaf fibers $E_{scler} = 2\text{-}23\,GPa$
- tensile tests on iris leaves $E_{rib} = 21\,GPa$
- volume fraction of ribs in the faces is 0.39

$$E_f = 0.39\,E_{rib} = 8.2\,GPa$$

G_c^* - assume tissue fresh (E parenchyma constant at high/normal turgor pressure)

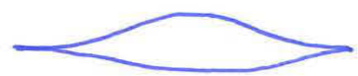
- data for $E_{parenchyma} = 0.5 - 6\,MPa$
- take $E_{parenchyma} \approx 4\,MPa$
- $G_c^* \sim \frac{1}{2} E_{parenchyma} = 2\,MPa$

Parenchyma Properties

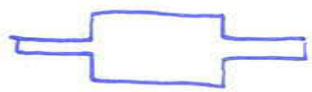
Plant material	Young's modulus, E^* or shear modulus, G^* (MPa)	Compressive strength, σ^*_{comp} (MPa)	Reference
Apple	$E^* = 0.31\text{-}3.46$	0.66	Oye et al., 2007
Apple	$E^* = 2.8\text{-}5.8$	0.25-0.37	Lin & Pitt, 1986
Apple	$G^* = 1\text{-}6$		Vincent, 1989
Potato	$E^* = 3.6$	1.3	Lin & Pitt, 1986
Potato	$E^* = 3.5$		Scanlon et al, 1996
Potato	$E^* = 5.5$	0.27	Hiller & Jeronimides, 1996
Potato	$G^* = 0.5\text{-}1$		Scanlon et al., 1996; 1998
Carrot	$E^* = 2\text{-}14$		Georget et al., 2003

Data for fresh, wet tissue, at normal turgor.

- using sandwich beam theory, can estimate P/δ (Table)
- calculation complicated by irregular thickness of core across section:



- rough attempt to account for this by dividing cross-section into sub-units



- found calculated P/δ overestimated measured P/δ by 16-83%
- agreement ok for the various approximations + estimates made.

<u>Strength of the leaf</u>

faces: $\sigma_{yf} = ?$

- previous tests on tensile strength of grass leaves found

$$\sigma_f \text{ (in MPa)} = 1.44\,(\underset{\substack{\uparrow \\ \text{Vol. fraction}}}{V_{\text{sclerenchyma}}} \times 100) + 1.53$$

- in Iris, ribs (assume all sclerenchymal are 80% dense + make up 40% of face

$$\sigma_{yf} = 1.44\,(0.8 \times 0.4 \times 100) + 1.53 = 47 \text{ MPa}$$

Iris Sandwich Analysis

Table 6.2 Beam bending results

Specimen	**1**	**2**	**3**	**4**
Measured beam stiffness, P/δ (N/mm)	0.66	0.54	0.41	0.25
Beam length, l (mm)	35	35	35	35
Face thickness, f (mm)	0.03	0.03	0.03	0.03
Maximum core thickness, c (mm)	4.63	3.31	2.49	1.51
Width, b (mm)	18	18	18	18
Flexural rigidity, D (Nm^2)	0.027	0.016	0.0096	0.0051
Bending compliance, $(\delta/P)_b$ (m/N)	5.29×10^{-4}	8.98×10^{-4}	1.49×10^{-3}	2.83×10^{-3}
Shear compliance, $(\delta/P)_s$ (m/N)	2.99×10^{-4}	3.83×10^{-4}	4.83×10^{-4}	6.3×10^{-4}
Calculated beam stiffness, P/δ (N/mm)	1.21	0.78	0.51	0.29
Calculated/measured beam stiffness	1.83	1.44	1.24	1.16

core : $\tau_c^* = ?$

- literature $\sigma^*_{tension} \approx 0.4$ MPa (parenchyma)
- expect $\tau_c^* \sim \frac{1}{2}\sigma^*_{tension} \sim 0.2$ MPa

- calculate strength of iris leaf in wind - cantilever, uniformly distributed load ($B_3 = 2$ $B_4 = 1$)
- calculate loads at base of leaf (M_{max}) : $t \sim 0.03$ mm $l \sim 600$ mm

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face yielding: $\frac{P_{fy}}{bc} = B_3\, \sigma_{yf} \left(\frac{t}{l}\right) = (2)\,(47\text{ MPa})\left(\frac{0.03}{600}\right) = 4.7$ kPa

face wrinkling: $\frac{P_{fw}}{bc} = 0.57\, B_3\, E_f^{1/3} E_c^{*2/3} (t/l) = (0.57)\,(2)\,(8.2\times10^9)^{1/3}(4\times10^6)^{2/3}\left(\frac{0.03}{600}\right)$
$= 2.8$ kPa

core shear: $\frac{P_{cs}}{bc} = B_4\, \tau_c^* = (1)\,(0.2\text{ MPa}) = 200$ kPa

- expect leaf failure by face wrinkling

- are iris leaves optimized?
  - $\delta_s / \delta_b = 0.22 - 0.57$ in specimens tested
  - in minimum weight design for given stiffness $\delta_s / \delta_b = 2$
  - but leaves have several functions beyond mechanical support:
    - photosynthesis; requires large surface area
    - fluid transport
  - difficult to quantify relative importance of each function to plant
  - engineering optimization not possible

Additional examples of sandwich structures in nature:

- marine "leaves" seaweed Durvillaea antarctica : fronds 12 m long
  honeycomb core
- bird skulls
  - if inner + outer face concentric - trabeculae oriented $\perp$ cortical shell
  - if " " " not " - " foam-like
  - larger birds have multiple sandwiches
    - since size of trabeculae relatively constant, this may allow larger core thickness
  - owl skull - asymmetry - improves hearing

Comparison of optimized sandwich plate with solid plate of same stiffness

- consider circular plate, radius R, simply supported around circumference, subject to a uniformly distributed load q [N/m²]
- central plate deflection is w
- if sandwich is optimized (based on analysis in book p 384)

$$\frac{\text{Mass}}{\pi R^2} = 1.49 \left(\frac{qR}{wE_s}\right)^{3/5} \rho_s R \qquad \text{(foam core)}$$

- equivalent solid plate:

$$\frac{\text{mass}}{\pi R^2} = 0.89 \left(\frac{qR}{wE_s}\right)^{1/3} \rho_s R$$

- taking the ratio:

$$\frac{M_{sandwich}}{M_{solid}} = 1.67 \left(\frac{qR}{wE_s}\right)^{0.27}$$

- consider bone sandwich, "foamed" trabecular core; $R = 100$ mm $P = 500$ N $(= q\pi R^2)$ $w = 1$ mm $E_s = 18$ GPa

$$\frac{qR}{wE_s} = 10^{-4} \qquad \frac{M_{sandwich}}{M_{solid}} = 14\%$$

⇒ optimized bone sandwich would be 14% weight of solid cortical panel

Additional examples (cont'd)

- cattle fish bone (not a fish - a mollusk ; not bone - $CaCO_3$)
- horseshoe crab shell.
- tortoise shell (Galapagos)

Durvillaea antarctica (New Zealand Seakelp)

Largest intertidal seaweed
Fronds up to 12m long
Fronds have gas-filled honeycomb-like core that provides buoyancy as well as flexural rigidity, maximizing surface area exposed to sunlight

http://en.wikipedia.org/wiki/File:Dried_bull_kelp_(Durvillaea_antarctica)_with_cross-section_showing_honeycomb_structure_IMG_102_1239.JPG

Bird Skulls

Alison Curtis

Photo of owl imprint in the snow removed due to copyright restrictions.

No footprints in the snow from mouse or vole; animal was under the snow

http://www.twincitiesnaturalist.com/2010/01/barred-owl-hunting-in-snow.html

Photo removed due to copyright restrictions. See Summit Post.
http://www.summitpost.org/disappearing-rabbit-trick/185785/c-186336

Rabbit tracks in snow
http://www.myconfinedspace.com/2006/12/21/owl-snowprint/

Cuttlefish bone
Mollusc shell ($CaCO_3$)

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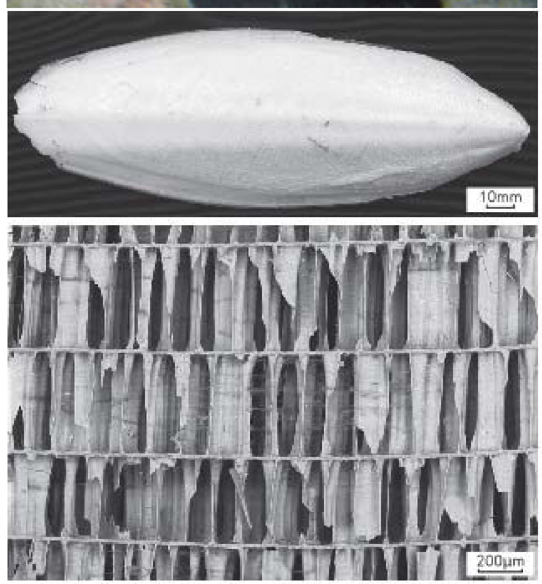

Gibson, L. J., M. Ashby, and B. A. Harley. *Cellular Materials in Nature and Medicine*. Cambridge University Press. © 2010. Figure courtesy of Lorna Gibson and Cambridge University Press.

Horseshoe Crab Shell

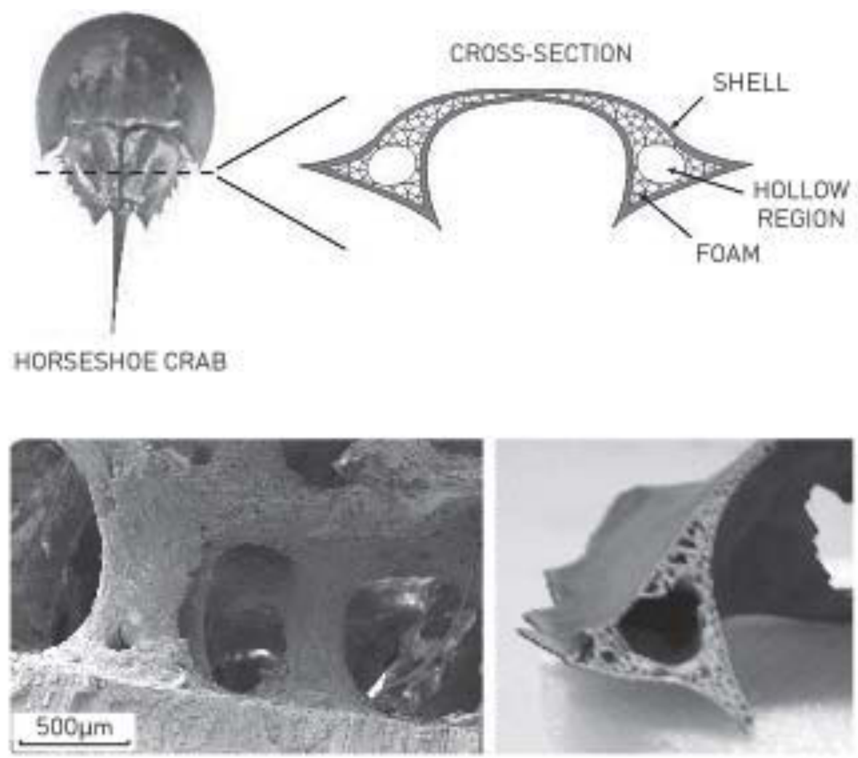

Figure 148: M. A. Meyers, P. -Y. Chen, et al. *Progress in Materials Science* 53 (2008): 1–206.

http://www.sciencedirect.com/science/article/pii/S0079642507000254

Meyers et al., 2008

Galapagos Tortoise Shell

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
Spring 2015