Lecture 5, 3.054

Honeycombs: Out of plane behavior

- Honeycombs used as cores in sandwich structures
  - carry shear load in $x_1 - x_3$ and $x_2 - x_3$ planes
- Honeycombs sometimes used to absorb energy from impact — loaded in x_3 direction
- Require out-of-plane properties
- Cell walls extend or contract, rather than bend
- Honeycomb much stiffer and stronger

Linear-elastic deformation

- Honeycomb has 9 independent elastic constants:
  - 4 in-plane
  - 5 out-of-plane

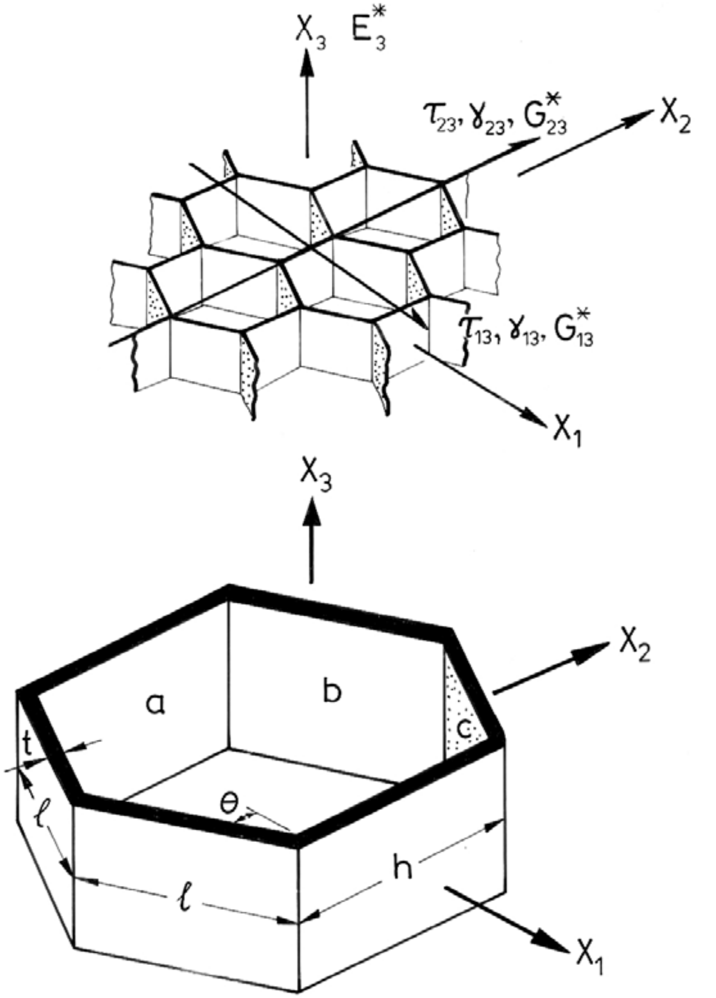
Young's Modulus, E_3^*

- Cell walls contract or extend axially
- E_3^* scales as area fraction of solid in plane perpendicular to x_3

$$\boxed{E_3^* = E_s(\rho^*/\rho_s) = E_s\left(\frac{t}{l}\right)\frac{h/l+2}{2(h/l+\sin\theta)\cos\theta}}$$

Notice: E_3^* $\quad \frac{t}{l}$ and $E_1^*,\ E_2^*$ $\quad \left(\frac{t}{l}\right)^3$ $\quad$ Large anisotropy

Out-of-Plane Properties



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Poisson ratios

- For loading in x_3 direction, cell walls strain by $\nu_s\,\epsilon_3$ in x_1, x_2 directions

$$\boxed{\nu_{31}^* = \nu_{32}^* = \nu_s} \qquad \text{(recall } \nu_{ij} = -\frac{\epsilon_j}{\epsilon_i}\text{)}$$

- ν_{13}^* and ν_{23}^* can be found from reciprocal relation:

$$\boxed{\frac{\nu_{13}^*}{E_1^*} = \frac{\nu_{31}^*}{E_3^*} \text{ and } \frac{\nu_{23}^*}{E_2^*} = \frac{\nu_{32}^*}{E_3^*}}$$

$$\therefore\ \nu_{13}^* = \frac{E_1^*}{E_3^*}\,\nu_{31}^* = \frac{c_1\left(\frac{t}{l}\right)^3 E_s\,\nu_s}{c_2\left(\frac{t}{l}\right)E_s} \approx 0 \quad \text{for small } \left(\frac{t}{l}\right)$$

Similarly, $\nu_{23}^* \approx 0$

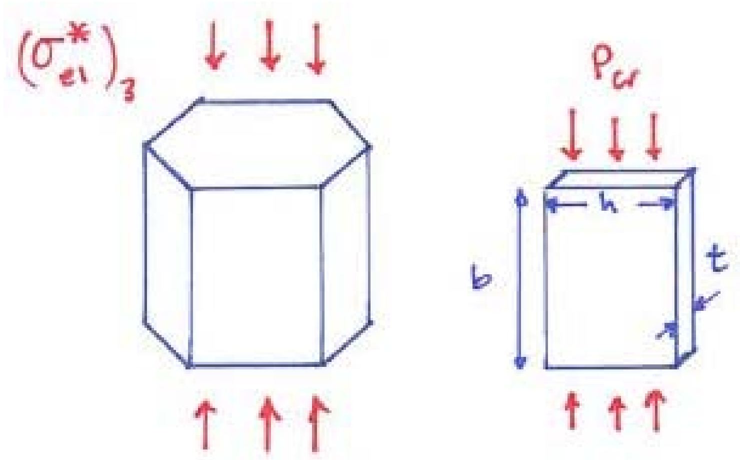
Shear moduli

- Cell walls loaded in shear
- But constraint of neighboring cell walls gives non-uniform strain in cell walls
- Exact solution requires numerical methods
- Can estimate as:

$$\boxed{G^*_{13} = G_s \left(\frac{t}{l}\right) \frac{\cos\theta}{h/l + \sin\theta} = \frac{1}{\sqrt{3}}\, G_s\, \frac{t}{l}} \quad \text{for regular hexagons } (= G^*_{23})$$

- Note linear dependence on $\left(\frac{t}{l}\right)$

Compressive strength: elastic buckling



- Plate buckling

$$P_{cr} = \frac{K E_s t^3}{(1-\nu_3^2)h} \qquad \text{also for l}$$

- K end constraint factor depends on stiffness of adjacent walls

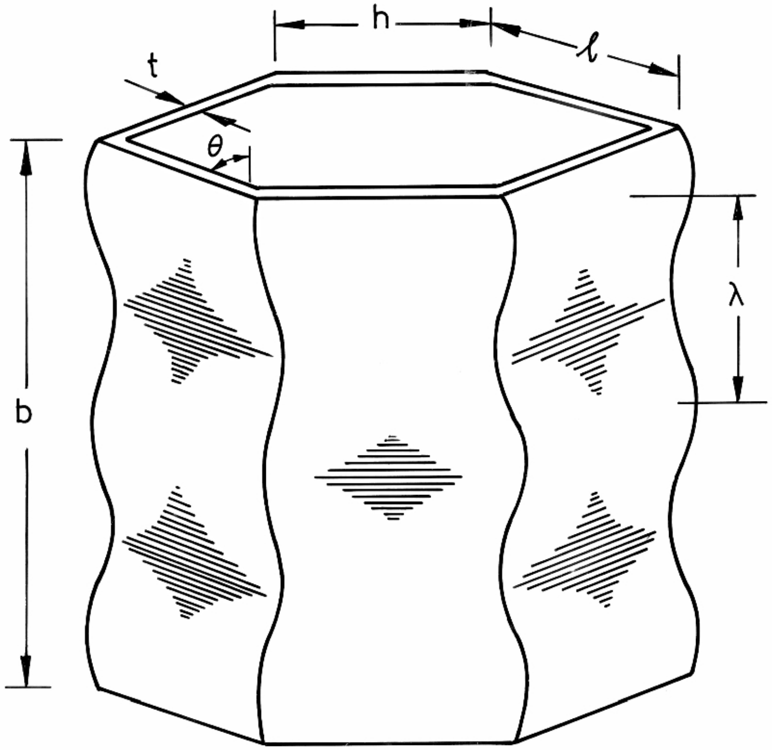
- If vertical edges are simply supported (free to rotate) and $b > 3l$: K=2.0
- If vertical edges are clamped and fixed: K=6.2
- Approximate $K \approx 4$

$$P_{total} = \sum P_{cr} \text{ for each wall } (2l + h \text{ for each cell})$$

$$\boxed{(\sigma^*_{el})_3 = \frac{E_s}{1-\nu_s^2}\left(\frac{t}{l}\right)^3 \frac{2(l/h+2)}{(h/l+\sin\theta)\ \cos\theta}}$$

- Regular hexagons $(\sigma^*_{el})_3 = 5.2 E_s \left(\frac{t}{l}\right)^3$
- Same form as $(\sigma^*_{el})_2$ but ~20 times larger

Out-of-Plane: Elastic Buckling



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Compressive strength: plastic collapse

- Failure by uniaxial yield $(\sigma^*_{pl})_3 = \sigma_{ys}\,(\rho^*/\rho_s)$
- **But**, in compression, plastic buckling usually precedes this
- Consider approximate calculation, simplified geometry   isolated cell wall
- Rotation of cell wall by π at plastic hinge
- Plastic moment $M_p = \frac{\sigma_{ys} t^2}{4}\,(2l + h)$ (note $2l + h$ instead of b as before, for loading in x_1 or x_2)
- Internal plastic work $= \pi M_p$
- External work done is $\frac{P\lambda}{2}$; λ is wavelength of plastic buckling $\approx l$;
  $P = \sigma_3(n + l\sin\theta)(2l\cos\theta)$

$$\therefore \quad \frac{P\lambda}{2} = \pi M_p$$

$$\sigma_3(h + l\sin\theta)(2l\cos\theta)\frac{l}{2} = \pi\frac{\sigma_{ys}t^2}{4}(2l + h)$$

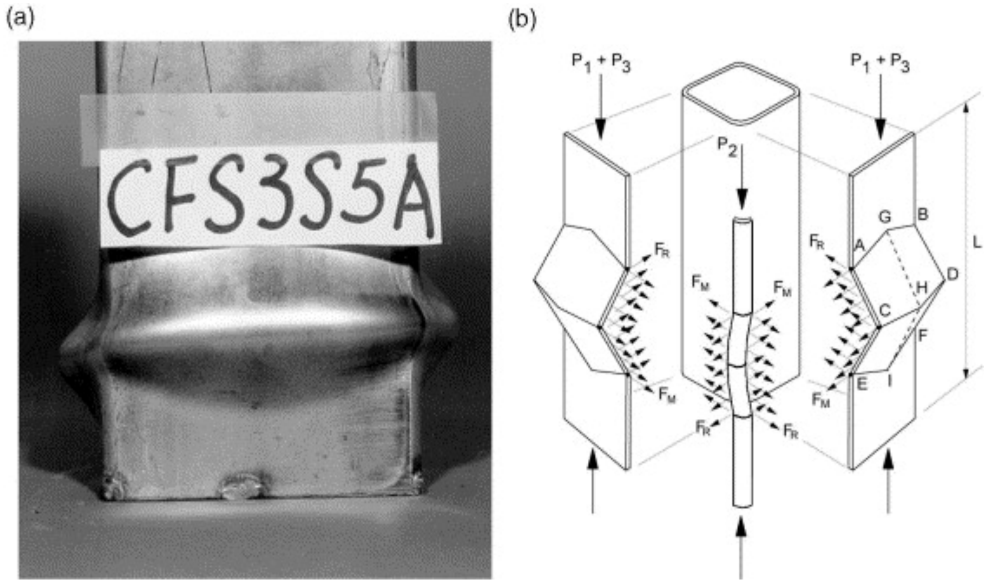
$$(\sigma^*_{pi})_3 \approx \frac{\pi}{4}\,\sigma_{ys}\left(\frac{t}{l}\right)^2\,\frac{(h/l + 2)}{(h/l + \sin\theta)\cos\theta}$$

Note: misprint in book equation before 4.115

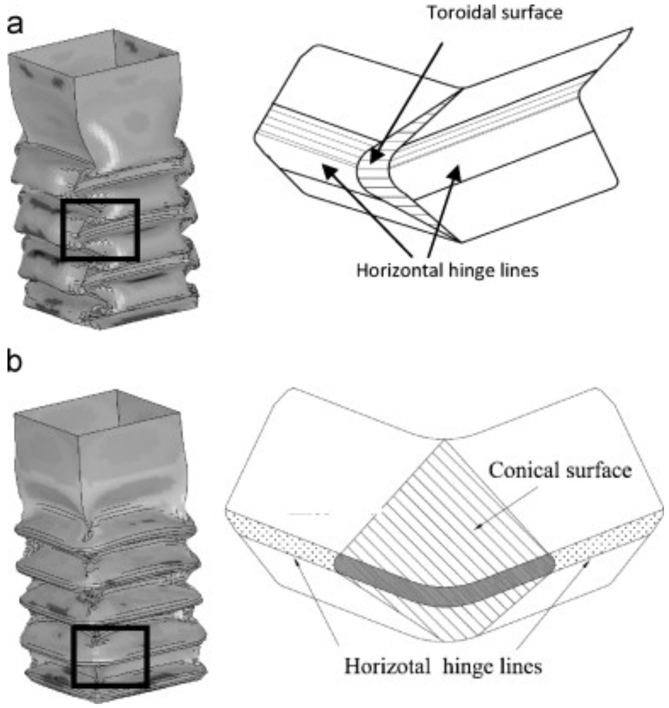
Regular hexagons: $(\sigma^*_{pi})_3 \approx 2\sigma_{ys}\left(\frac{t}{l}\right)^2$

Exact calculation for regular hexagons: $(\sigma^*_{pi})_3 = 5.6\,\sigma_{ys}\left(\frac{t}{l}\right)^{5/3}$

Out-of-Plane: Plastic Collapse



Source: Zhao X. L., B. Han, et al. "Plastic Mechanism Analysis of Concrete-Filled Double-skin (SHS Inner and SHS Outer) Stub Columns." *Thin-Walled Structures* 40 (2002): 815-33. Courtesy of Elsevier. Used with permission.

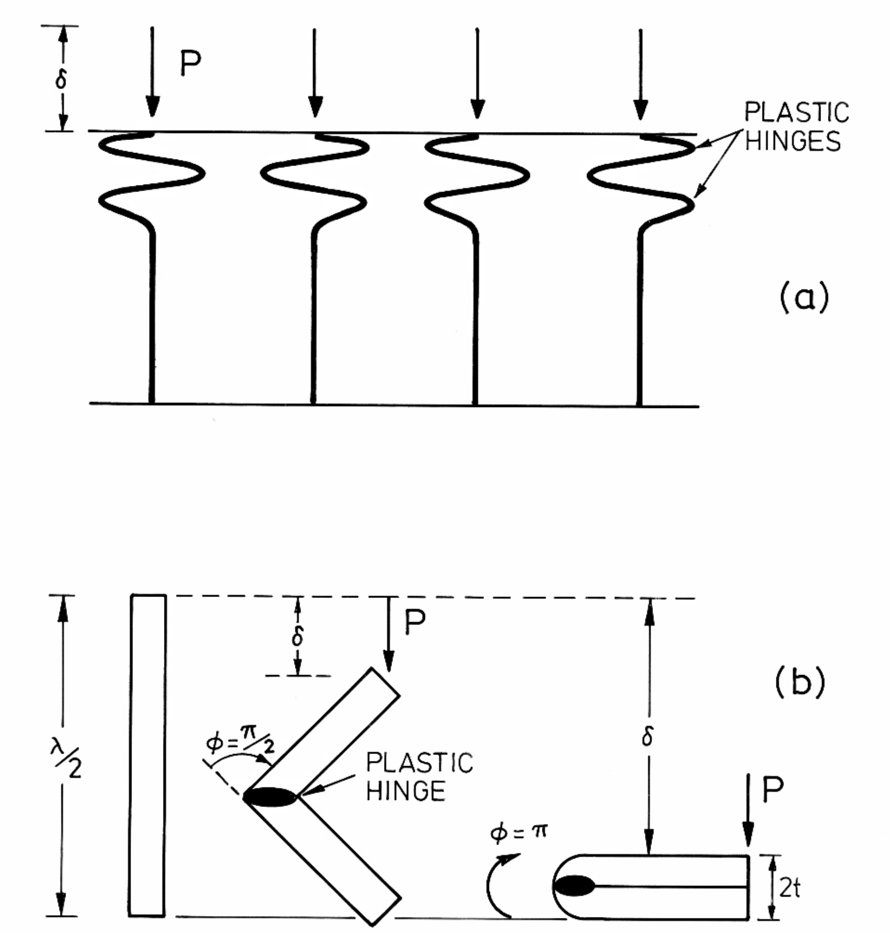


Source: Najafi, A., and M. Rais-Rohani. "Mechanics of Axial Plastic Collapse in Multi-cell, Multi-corner Crush Tubes." *Thin-Walled Structures* 49 (2011): 1-12. Courtesy of Elsevier. Used with permission.

Zhao XL,, Han B and Grzebieta RH (2002) Thi-Walled Structures **40**, 815-533

Najafi A and Rais-Rohani M (2011) Thin-Walled Structures **49**, 1-12

Out-of-Plane: Plastic Collapse



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Out-of-plane brittle fracture (tensile failure)

- Defect-free sample, walls see uniaxial tension

$$(\sigma_f^*)_3 = (\rho^*/\rho_s)\sigma_{fs} = \frac{h/l+2}{2(h/l+\sin\theta)\cos\theta}\Big(\frac{t}{l}\Big)\sigma_{fs}$$

- If cell walls cracked ($a >> l$) and crack propagates in plane normal to x_3:
  - Toughness, $G_c^* = (\rho^*/\rho_s)G_s$
  - Fracture toughness, $K_{Ic}^* = \sqrt{E^* G_c^*} = \sqrt{(\rho^*/\rho_s)E_s(\rho^*/\rho_s)G_{cs}} = (\rho^*/\rho_s)K_{Ics}$

Out-of-plane: brittle crashing

σ_{cs} = compressive strength of cell wall

$(\sigma_{cr}^*)_3 = (\rho^*/\rho_s)\sigma_{cs}$ brittle materials: $\sigma_{cs} \approx 12\,\sigma_{fs}$

$\approx 12(\rho^*/\rho_s)\sigma_{fs}$

3.054 / 3.36 Cellular Solids: Structure, Properties and Applications
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