3.032 Problem Set 7
Fall 2007

Due: **<u>Start</u>** of Lecture, Friday 12.07.07

1. If one's appendix becomes infected with bacteria, it can rupture or perforate. The contents of the infected appendix then leak into the abdomen, leading to periappendiceal abscess (a collection of infected pus in the abdomen and pelvis), which is as bad as it sounds and can be fatal.

Although perforation of the appendix requires immediate treatment whether fast or slow, a leak-before-break condition is preferable because this gives more time for the patient to have the slowly leaking appendix removed via surgery.

Fig. 1: Human appendix (uninfected) is a cylindrical appendage extending from the cecum of the large intestine. The typical size of an appendix is 10 cm long x 1 cm in diameter, with a wall thickness of about 1 mm. At rupture, the internal pressure from the infection reaches about 1 MPa.

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(a) Idealize the appendix wall material as an isotropic, elastic-to-brittle solid, and determine the critical crack size *a* of the appendix wall required for the appendix to leak-before-breaking. Let 2a = 2c, assuming a semicircular through-thickness "crack" in the appendix wall.

(b) Comment on whether this prediction is reasonable, vis a vis the size of the appendix and the relative infrequency of ruptured appendices.

(c) Explain how you would determine the critical wall thickness *t* of other organ "pressure vessels" such as the bladder, if you knew a pre-existing crack size (say, from a surgical incision) and needed to determine the critical thickness of the organ wall for a specific magnitude of internal organ pressure *p*.

(d) Assuming appendix rupture really was well described by brittle fracture. Prof. X's appendix burst at a critical crack length *a* = 0.1 mm, under an internal pressure that was 5 MPa just prior to catastrophic failure. What were the continuum mechanical properties K_{IC}, G_{IC}, and J_{IC} of Prof. X's appendix wall material, which is mostly smooth muscle?

(e) Why is the appendix poorly described by Griffith's fracture criterion?

2. The stresses around a crack tip are "magnified" because the crack faces are displaced a distance **u** inside the material, creating a strain $\boldsymbol{\varepsilon}$(r, θ, a) and thus a stress $\boldsymbol{\sigma}$(r, θ, a) inside the material. This is analogous to the stresses created by a dislocation inside a material, though the symmetry breaking is different

The stresses around the crack tip under plane strain conditions are given by

$\sigma_{xx} = \{K_I/[2\pi r]^{1/2} \cos\theta/2\}\ (1 - \sin[\theta/2] \sin[3\theta/2])$

$\sigma_{yy} = \{K_I/[2\pi r]^{1/2} \cos\theta/2\}\ (1 + \sin[\theta/2] \sin[3\theta/2])$

$\sigma_{xy} = \{K_I/[2\pi r]^{1/2} \cos\theta/2\}\ (\sin[\theta/2] \cos[3\theta/2])$

(a) What does "plane strain" mean in terms of the dimensions of the material that contains the crack and the way in which the crack is loaded?

(b) Determine the other normal stress σ_{zz} and the shear stresses σ_{xz} and σ_{yz} in terms of these stresses and any other required mechanical properties of the material, remembering that linear elastic fracture mechanics idealizes the material as an isotropic elastic continuum.

(c) Graph the largest of these stress components as a function of distance from the crack tip. Here, you can normalize by any quantities you do not know, such as the magnitude of applied stress.

(d) The radius of the plastic zone around a crack tip r_p is given by the distance from the crack tip over which the stress exceeds the yield stress of the material. Determine the size of this plastic zone $r_p(\sigma, a, \sigma_y)$ by evaluating the crack tip stresses σ_{ij} at $\theta = 0$.

(e) Now compare the size of this plastic zone for a crack of length a = 1 mm under a Mode I stress σ = 100 MPa in Au, Cu, W, Si, and amorphous SiO_2.

3. From the literature, determine the Young's elastic modulus E, yield strength σ_y and fracture toughness K_{IC} of any three materials of interest to you.

(a) Graph σ_y vs. K_{IC}, E vs. σ_y, and E vs. K_{IC}. Comment on the observed trends (noting that there may be no clear trend in some cases).

(b) Given these trends, how would you design a general microstructure for which the application demanded that the material be stiff, strong, and tough. Be as specific as possible, and feel free to draw this schematic microstructure to illustrate your reasoning.

(c) Figure 2 shows a scanning electron micrograph of a fracture surface. Is this of a metal, polymer, or ceramic, and is it indicative of ductile fracture, brittle intergranular fracture, or brittle transgranular fracture?

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Fig. 2: SEM fractograph of material. Scalebar= 100 um.

Fig. 3: Fatigue striations evident in SEM micrograph of 302 stainless steel spring that has fractured.

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4. Figure 3 shows the fracture surface of a 302 stainless steel spring. This spring was under a cyclic stress between 0 and 100 MPa at a frequency of 1 kHz. We can assume that the initial crack size *a* was at the limit of the resolution of an optical microscope, with which the spring

was inspected before use. Young's elastic modulus E, yield strength σ_y and fracture toughness K_{IC} of this steel are 210 GPa, 500 MPa, and 100 MPa $m^{1/2}$, respectively.

(a) Calculate the crack growth rate during steady-state crack propagation, da/dN. Compare this with the average da/dN you measure from the fractograph in Fig. 2b.

(b) Assuming the crack was already at the critical crack length to propagate at this applied stress, how many minutes was the spring in use before fatigue failure? Note that failure time is a product of the number of cycles to failure and the cyclic operating frequency of the structure.