

Lecture 18
The P-N Junction (The Diode).

Today:

1. Joining *p*- and *n*-doped semiconductors.
2. Depletion and built-in voltage.
3. Current-voltage characteristics of the *p-n* junction.

Questions you should be able to answer by the end of today's lecture:

1. What happens when we join *p*-type and *n*-type semiconductors?
2. What is the width of the depletion region?
How does it relate to the dopant concentration?
3. What is built-in voltage? How to calculate it based on dopant concentrations?
How to calculate it based on Fermi levels of semiconductors forming the junction?
4. What happens when we apply voltage to the *p-n* junction?
What is forward and reverse bias?
5. What is the current-voltage characteristic for the *p-n* junction diode?
Why is it different from a resistor?

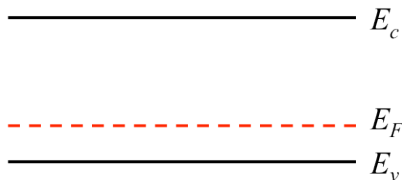
From previous lecture we remember:

p-type material at equilibrium

$$p_v \approx N_A$$

$$n_c \approx \frac{n_i^2}{N_A}$$

$$\mu_p = \mu_i - k_b T \ln\left(\frac{N_A}{n_i}\right)$$



n-type material at equilibrium

$$n_c \approx N_D$$

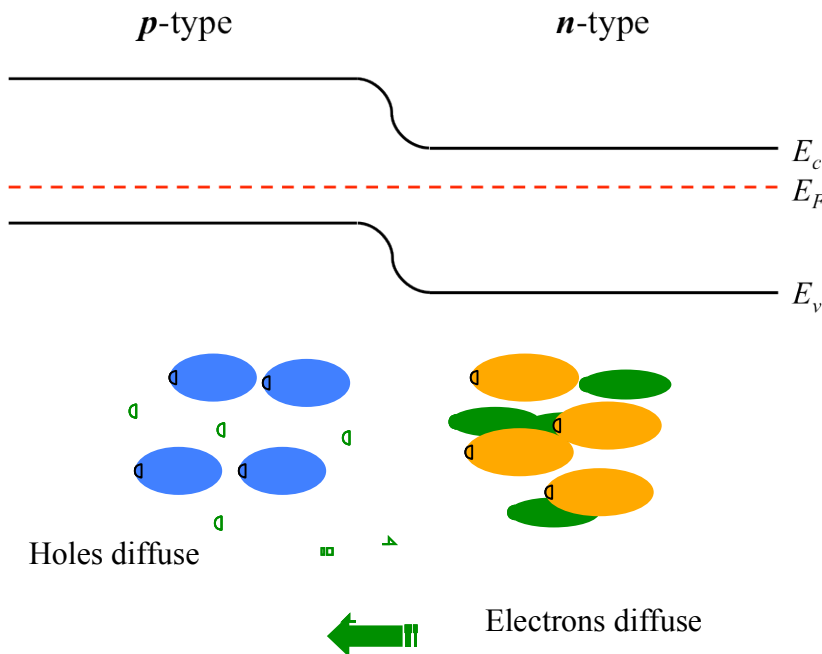
$$p_v \approx \frac{n_i^2}{N_D}$$

$$\mu_n = \mu_i + k_b T \ln\left(\frac{N_D}{n_i}\right)$$



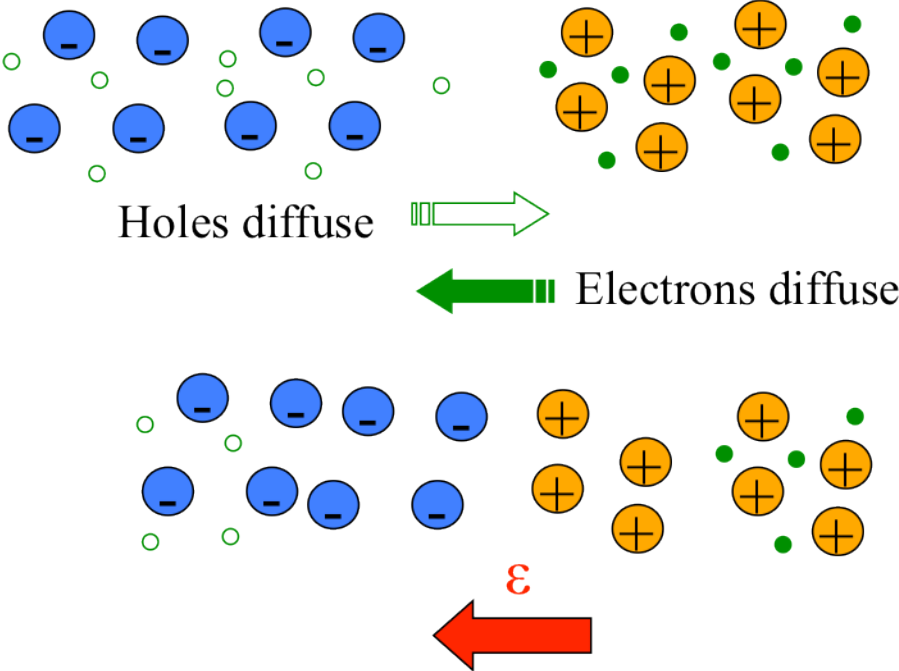
What happens when you join **p**-doped and **n**-doped pieces of semiconductor together?

When materials are put in contact the carriers flow under driving force of diffusion until chemical potential on both sides equilibrates, which would mean that the position of the Fermi level must be the same in both **p** and **n** sides. This results in band bending:



The electrons will diffuse into p-type material where they will recombine with holes (fill in holes). And holes will diffuse into the n-type materials where they will recombine with electrons.

This means that eventually in vicinity of the junction all free carriers will be depleted leaving stripped ions behind, which would produce an electric field across the junction:



The electric field results from the deviation from charge neutrality in the vicinity of the junction.

$$\epsilon = \frac{1}{q} \frac{dE_c}{dx}$$

Here dE_c is the change in the energy of the conduction band across the junction.

A steady-state balance of carriers is achieved at the junction where diffusive flux of the carriers is balanced by the drift flux.

The loss of charge neutrality at the junction can be also expressed in terms of the potential, which is referred to as built-in voltage V_{bi} .

Using the density of the ionized donors at the junction (also referred to as space charge) $\rho(x)$ we can calculate the built-in voltage and the electric field at the junction.

$$\epsilon = \int \frac{\rho(x)}{\epsilon_r \epsilon_0} dx, \text{ where } \epsilon_r \epsilon_0 \text{ is a dielectric constant.}$$

$$V_{bi} = - \int \epsilon(x) dx$$

The region in the vicinity of the junction, which has been depleted of the free charge carriers is called “depletion region”. The width of the depletion region is:

$$W = \sqrt{\frac{2\epsilon_r \epsilon_0 V_{bi}}{q} \frac{N_A + N_D}{N_A N_D}}$$

The width of the depleted **n**-type region (which is left positively charged):

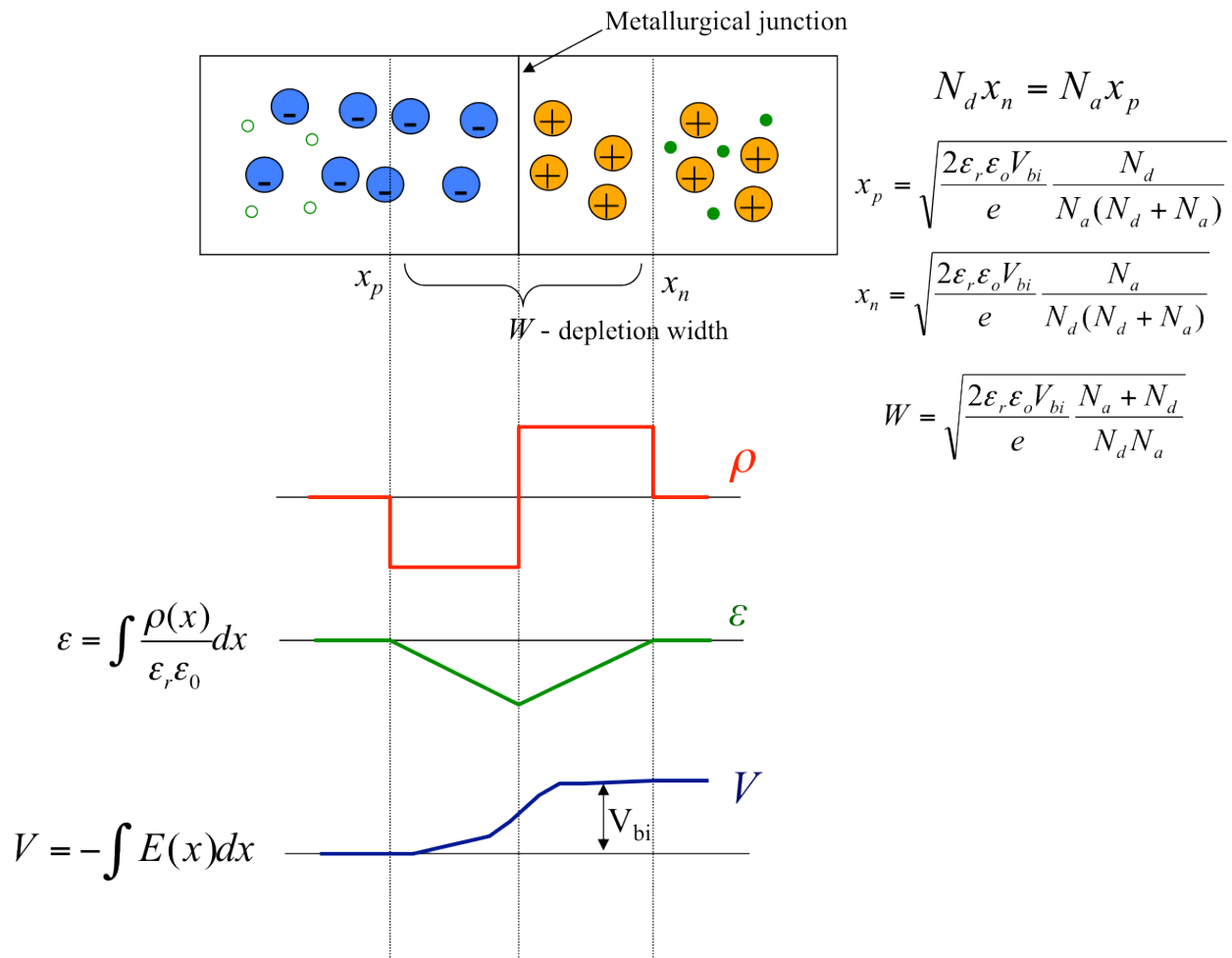
$$x_n = \sqrt{\frac{2\epsilon_r \epsilon_0 V_{bi}}{q} \frac{N_A}{N_D (N_A + N_D)}}$$

The width of the depleted **p**-type region (which is left negatively charged):

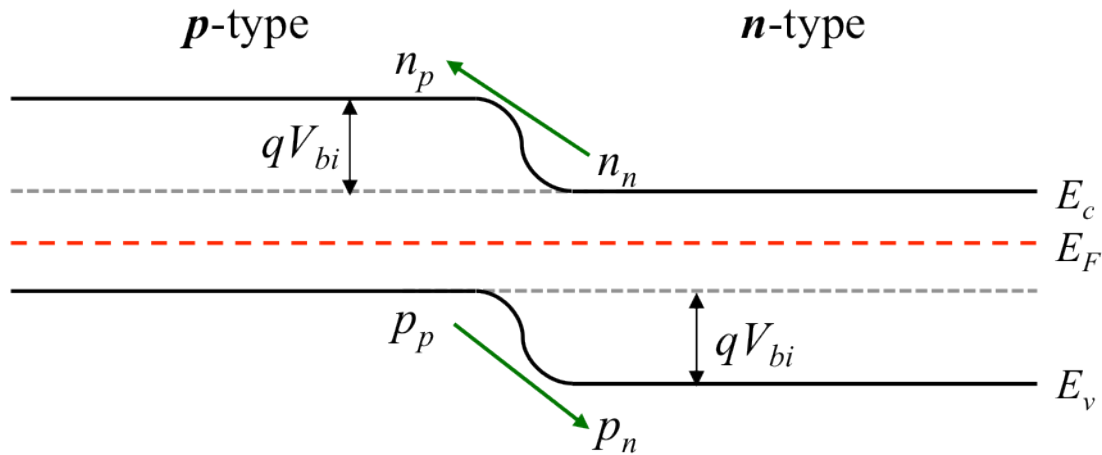
$$x_p = \sqrt{\frac{2\epsilon_r \epsilon_0 V_{bi}}{q} \frac{N_D}{N_A (N_A + N_D)}}$$

Consequently: $N_A x_p = N_D x_n$

This means that at the junction higher doped material will have narrower depletion region and lower doped material will have wider depletion region.



What is the built-in voltage V_{bi} ?



Built-in voltage is simply the difference of the Fermi levels in p- and n-type semiconductors before they were joined.

$$qV_{bi} = E_{Fn} - E_{Fp}$$

$$E_{Fp} = E_{Fi} - k_B T \ln\left(\frac{N_A}{n_i}\right)$$

$$E_{Fn} = E_{Fi} + k_B T \ln\left(\frac{N_D}{n_i}\right)$$

Then we can express the built-in voltage in terms of the doping concentrations:

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

Electrons that move from **n**-type to **p**-type or holes moving in the opposite direction are called minority carriers (holes inside **n**-type or electrons in **p**-type). Using equations above we can show that qV_{bi} is the barrier to minority carrier injection:

$$p_n = p_p e^{\frac{-qV_{bi}}{k_B T}}$$

$$n_p = n_n e^{\frac{-qV_{bi}}{k_B T}}$$

Example: Doping GaAs

Figure removed due to copyright restrictions. Fig. 4.14: Unknown source.

$$V_{bi} = \frac{k_B T}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right)$$

$$N_{Zn(A)} = 10^{16} \frac{1}{\text{cm}^3}$$

$$N_{Te(D)} = 10^{16} \frac{1}{\text{cm}^3}$$

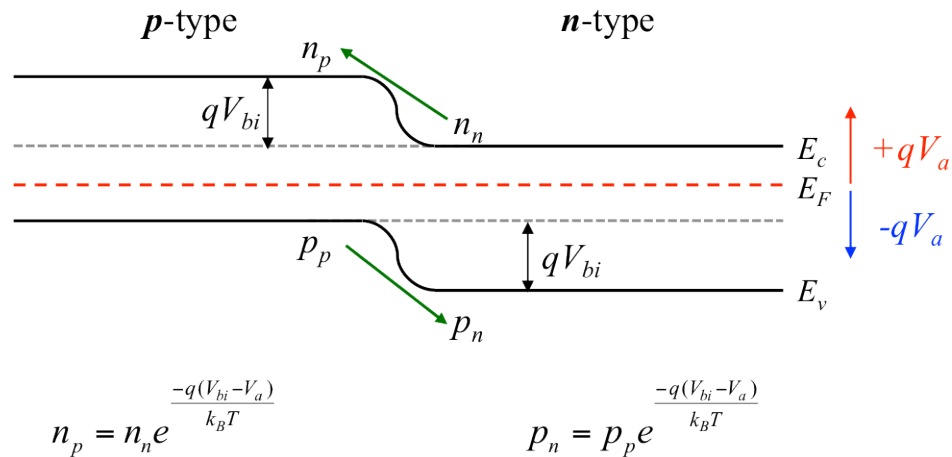
$$n_{i(GaAs\ 300K)}^2 = 10^{12}$$

$$V_{bi} = 0.025 \ln \left(\frac{10^{32}}{10^{12}} \right) = 1.15V$$

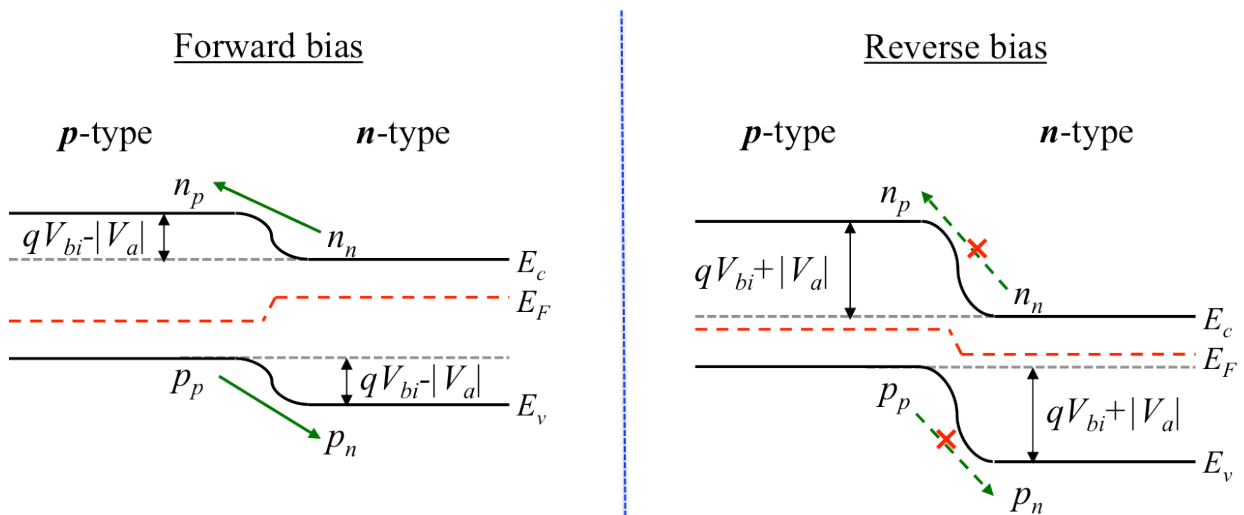
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Fig. 2.20: Pierret, Robert F. *Semiconductor Fundamentals*. 2nd ed. Prentice Hall, 1988.

Effect of Applied Voltage - Bias:

When we apply forward bias (positive voltage to **p**-type, negative voltage to **n**-type) it effectively reduces the built-in voltage. When we apply reverse bias (positive voltage to **n**-type, negative voltage to **p**-type) it effectively increases the built-in voltage.



- Applying a potential to the ends of a diode does NOT increase current through drift rather it lowers the potential barrier to diffusion.
- The applied voltage upsets the steady-state balance between drift and diffusion, which unleashes the flow of diffusion current.
- Since the conduction through the junction happens via minority carriers, the p-n junction is called a “Minority carrier device.”
- Forward bias decreases depletion region and also increases diffusion current exponentially.
- Reverse bias increases depletion region, and in ideal case there is no current flow.



If we solve the minority carrier drift and diffusion equations:

$$J_n = q\mu_n n \mathcal{E} - qD_n \frac{dn}{dx}$$

$$J_p = q\mu_p p \mathcal{E} - qD_p \frac{dp}{dx}$$

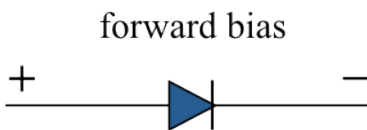
Where D_n and D_p are electron and hole minority carrier diffusion constants.

We can find the total dependence of the current through the junction on the applied bias voltage:

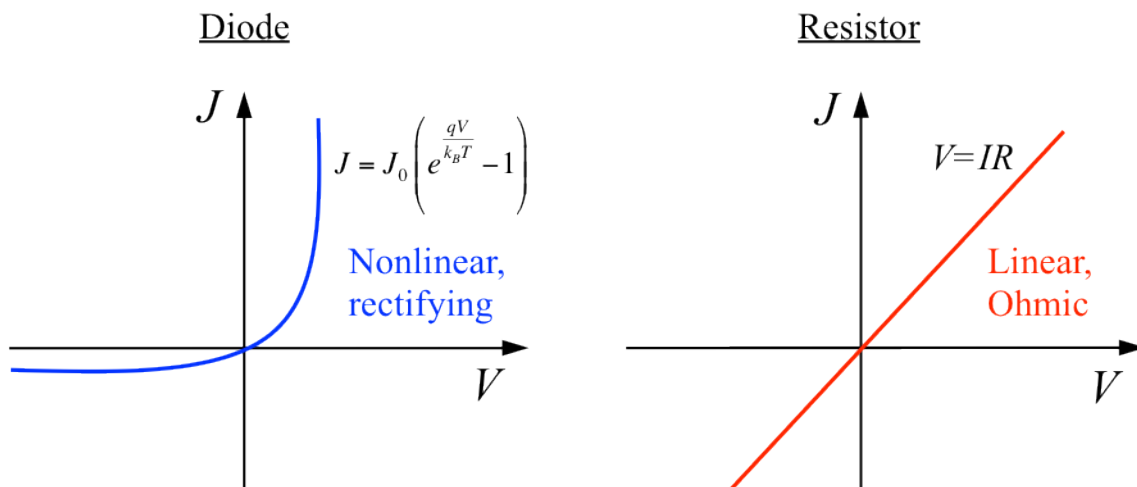
$$J = q \left(\frac{D_n}{L_n} \frac{n_i^2}{N_A} + \frac{D_p}{L_p} \frac{n_i^2}{N_D} \right) \left(e^{\frac{qV_a}{k_B T}} - 1 \right) = J_s \left(e^{\frac{qV_a}{k_B T}} - 1 \right),$$

Here $L_{n,p} = \sqrt{D_{n,p} \tau_{n,p}}$ are diffusion lengths for the minority carriers.

Since the **p-n** junction demonstrates such a unipolar (rectifying) response to the applied voltage it is called a **p-n** diode and is denoted in circuit diagrams as a following symbol:



The current voltage (IV) characteristic for the diode is rectifying and is very different from that for a resistor.



Ideal Diode I-V Characteristics

$$J = J_n + J_p = J_s \left(e^{\frac{qV}{kT}} - 1 \right)$$

Figure removed due to copyright restrictions. See Fig. 15.1-18(c): Saleh, Bahaa E. A., and Malvin Carl Teich. *Fundamentals of Photonics*. 2nd ed. Wiley, 2007.

What kind of devices can we build using p-n junctions?

- **P-N junctions under applied voltage:** diodes, transistors, light-emitting devices, and lasers.
- **P-N junctions under illumination:** solar cells, photodetectors.

Before we can design p-n junction devices that couple electricity to light and vice versa we need to understand concepts of carrier generation and recombination.

Carrier generation. Photovoltaic cells.

The two most common mechanisms for carrier generation in semiconductors are thermal generation and photogeneration. Thermal generation is a standard mechanism for promotion of the electrons from valence to conduction bands, which is just a result of Fermi distribution broadening at higher temperatures.

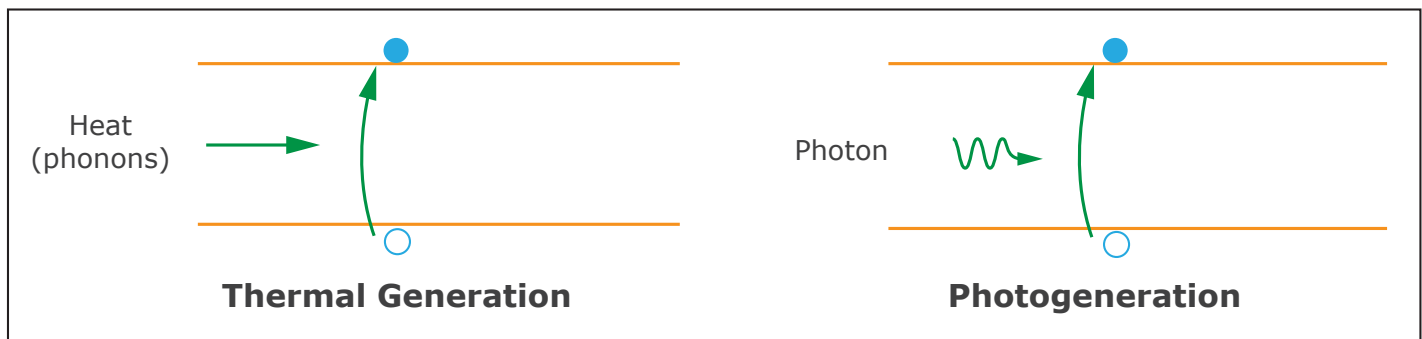
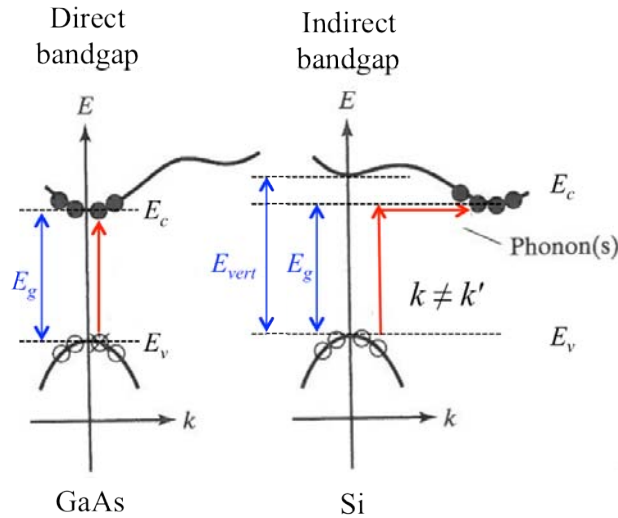


Image by MIT OpenCourseWare.

The second mechanism – photogeneration is more exciting as it involves an interaction of an electron with a photon, which results in electron being promoted into conduction band leaving behind a hole. **In other words, absorption of a photon by a material results in a generation of an electron-hole pair.**

Let's focus on photogeneration and its application to photodetectors and photovoltaics.

The photogeneration is a direct consequence of absorption of a photon. The absorption rate is highly dependent on the type of semiconductor.



• **Direct bandgap semiconductors:**

The top of the valence band aligns with the bottom of the conduction band in k -space. In this case the material can absorb any photon with the energy equal or larger than the bandgap.

• **Indirect bandgap semiconductors:**

The top of the valence band is shifted from the bottom of the conduction band in k -space. In these materials photons with energies between the bandgap and the vertical gap can only be absorbed in the presence of a lattice vibration (a phonon), which can donate its momentum towards the optical transition.

In direct bandgap semiconductors:

$$E_c^{electron} = E_v^{electron} + \hbar\omega, \quad \hbar\omega \geq E_g$$

In indirect bandgap semiconductors:

$$\left\{ \begin{array}{l} E_c^{electron} = E_v^{electron} + \hbar\omega + E_{phonon}, \quad E_g \leq \hbar\omega \leq E_{vertical} \\ \hbar k_c^{electron} = \hbar k_v^{electron} + p_{phonon} \end{array} \right.$$

and

$$E_c^{electron} = E_v^{electron} + \hbar\omega, \quad \hbar\omega \geq E_{vertical}$$

Figure removed due to copyright restrictions. Figs. 4-5: Kittel, Charles. *Introduction to Solid State Physics*. 8th ed. Wiley, 2004, p. 202.

Photogeneration rate G [electrons/sec] is proportional to the number of incident photons, which are generally measured as photon flux ϕ (photons/sec). In a simplest approximation we can say that generation rate G can be found as:

$$G \sim \frac{\text{absorbed photons}}{\text{sec}} = \alpha \cdot \phi, \text{ where } \alpha \text{ is absorption coefficient and } \phi \text{ is a photon flux.}$$

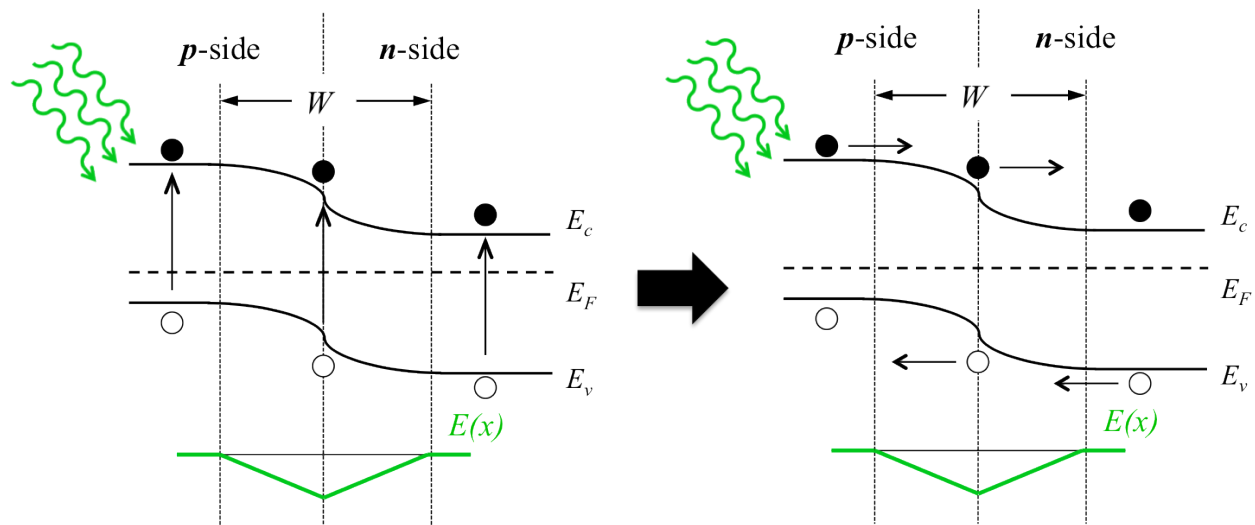
In reality, however, there are many factors (geometry, interactions with phonons etc.) contributing to the efficiency of conversion of photon into electron-hole pairs and the generation rate can be written as:

$$G = \alpha \cdot \phi \cdot \eta, \text{ where } \eta \text{ is the efficiency coefficient, which is generally measured experimentally.}$$

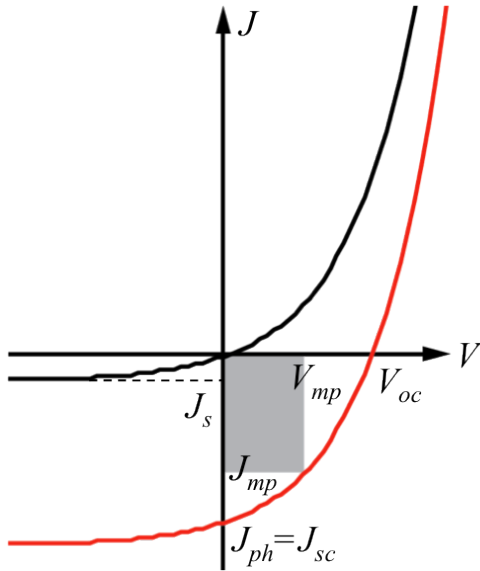
So what happens when light with the energy above the bandgap shines onto the p-n junction diode?

Outside the depletion region photogenerated electrons can easily fall back onto valence band or “recombine” with their holes unless they diffuse into the depletion region.

Remember that within the depletion region there is a built-in potential, or an electric field, which would immediately sweep the photogenerated holes and electrons into opposite directions. This means that the carriers generated within the depletion region or those that have diffused into the depletion region will be pushed by the field into the bulk of the material (holes into the p -side and electrons into the n -side).



- When electron-hole pairs approach the junction they are pulled apart by the built-in field.
- Electrons are pushed into the n -side and holes are pushed into the p -side.



- If we connect p-side to n-side (i.e. make a short circuit), then these carriers will flow at zero applied voltage (just under built-in voltage). This means we will observe short-circuit current called photocurrent J_{ph} .
- If we isolate the contacts the carriers accumulating on p and n sides would eventually lead to a potential build-up (effectively lowering built-in voltage), which would increase the dark current through the diode cancelling the photocurrent $J_{ph}=J_{sc}$. This potential is called open-circuit voltage V_{oc} .
- If we plot current-voltage characteristics for the p-n diode under illumination it will look shifted down by $J_{ph}=J_{sc}$.

Current-voltage characteristic for a diode:

$$J_{dark} = J_s \left(e^{\frac{eV_a}{k_B T}} - 1 \right)$$

Current-voltage characteristic for a diode under illumination:

$$J_{light} = J_{dark} - J_{ph} = J_s \left(e^{\frac{eV_a}{k_B T}} - 1 \right) - J_{ph}$$

Solving for voltage in the open-circuit regime we find V_{oc} :

$$J_{oc} = J_{dark} - J_{ph} = J_s \left(e^{\frac{eV_a}{k_B T}} - 1 \right) - J_{sc} = 0 \Rightarrow V_{oc} = \frac{k_B T}{e} \ln \left(\frac{J_{sc}}{J_s} + 1 \right)$$

Characteristics of solar cells:

1. Fill Factor (FF).

How do we extract energy out of a solar cell? The only way to do it currently is by attaching a load to it (the simplest example is a resistor).

It turns out that the resistance of the load is crucial to how much power one can possibly extract from a solar cell.

Recall that for the resistor the power is given by the following expression: $P = J^2 \cdot R = V \cdot J$

If we plot it in the I-V plot, the power will be the area of the rectangle formed by the values of I and V. Now take a closer look at the I-V characteristic of the solar cell. The only power we can extract would be a rectangle that fits between the V_{oc} and J_{sc} . There are a lot of those

rectangles but we can find the one that maximizes the area, i.e. maximizes the extracted power:

$$P_{\max} = J_{mp} V_{mp}$$

Hence, the optimum load has the resistance that satisfies: $R_{mp} = \frac{V_{mp}}{I_{mp}}$

Obviously, we would like to be able to extract as much power as possible from a solar cell, hence it is beneficial to have the space between the V_{oc} and J_{sc} to be as close to the rectangle between the V_{mp} and I_{mp} as possible. The parameter that defines that is called a fill factor (FF):

$$FF = \frac{V_{mp} J_{mp}}{V_{oc} J_{sc}}$$

2. Power conversion efficiency:

The power conversion efficiency is simply the ratio of the extracted power to the power of the incident light (optical power):

$$\eta_{power} = \frac{V_{mp} J_{mp}}{P_{optical}} = FF \frac{V_{oc} J_{sc}}{P_{optical}}$$

3. Quantum efficiency:

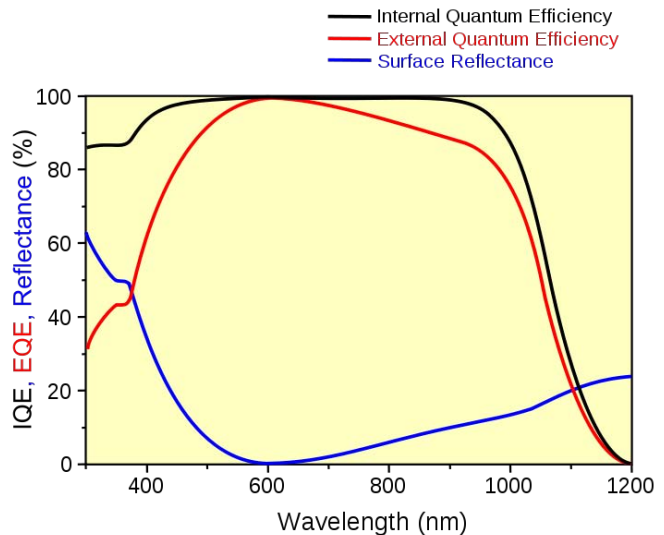
While power conversion efficiency is used in the industrial setting, solar cell designers and researchers are generally interested in the intrinsic behavior of the solar cell, i.e. how many electrons does the solar cell produce per one incident photon.

Internal quantum efficiency (IQE) is the property of the solar cell design rather than the material used to fabricate the cell, i.e. IQE is normalized to the material absorption:

$$IQE = \eta_{int} = \frac{\text{electrons out}}{\text{photons absorbed}}$$

External quantum efficiency (EQE) combines the properties of the cell design and the ability of the material to absorb:

$$EQE = \eta_{ex} = \frac{\text{electrons out}}{\text{photons in}}$$



The plot above demonstrates the difference between IQE and EQE. EQE takes into account material reflectance spectrum, i.e. it takes into account the fact that material does not absorb a number of photons but rather reflects them.

EQE is a standard characteristic generally used in scientific papers to describe the quality of a novel solar cell material or design as it reflects all the material and design properties (absorption, photon-to-electron efficiency, recombination processes, electron extraction efficiency etc.).

In order to calculate find EQE we generally use a light source with a known spectrum and power (usually “Solar Simulator” – a light source simulating solar spectrum) we then record IV-characteristics for the solar cell under illumination and compare it to the IV-characteristic without the light.

$$EQE = \eta_{ex} = \frac{\text{electrons out}}{\text{photons in}} = \frac{\text{electrons/s}}{\text{photons/s}} = \frac{\text{current}}{\text{photon flux}}$$

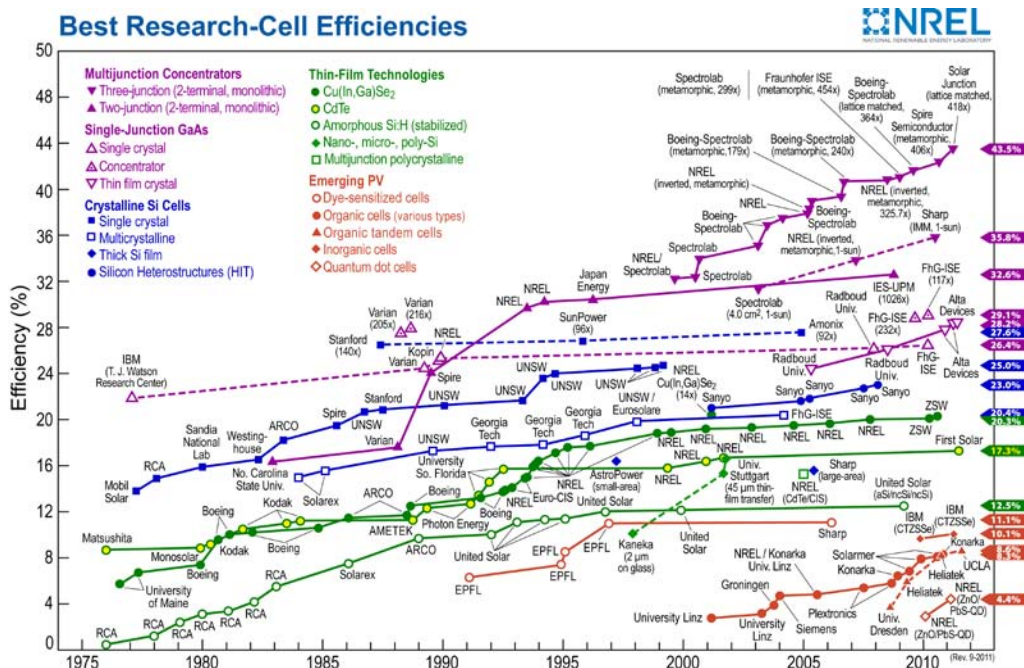
If we know the light power P [W] at a wavelength λ , then flux ϕ is:

$$\phi = \frac{P}{\text{photon energy}} = \frac{P}{ch/\lambda}, \text{ where } h = 2\pi\hbar \text{ is the Planck constant and } c \text{ is the speed of light.}$$

If we know the collected current I [A], then the number of electrons is: $n = \frac{I}{e}$

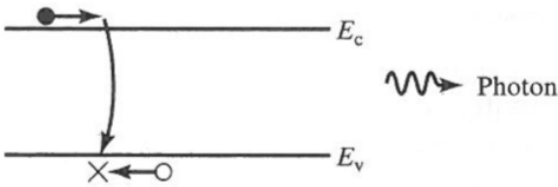
Then we can calculate the EQE of the solar cell:

$$EQE = \eta_{ex}(\lambda) = \frac{n}{\phi(\lambda)} = \frac{I/e}{P\lambda/ch} = \frac{Ich}{eP\lambda}$$

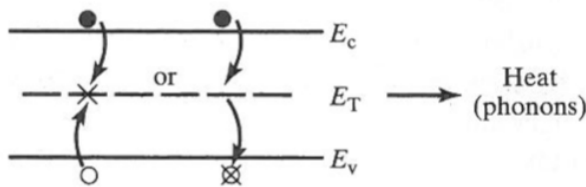


NREL compilation of best research solar cell efficiencies. (This image is in the public domain. Source: [Wikimedia Commons.](#))

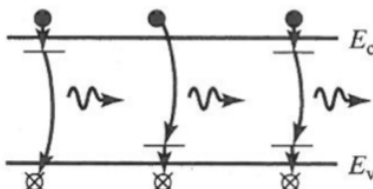
Recombination processes decreasing the efficiency of solar cells:



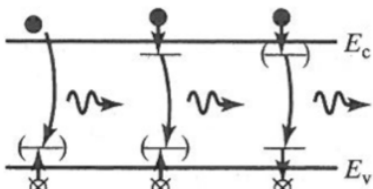
(a) Band-to-band recombination



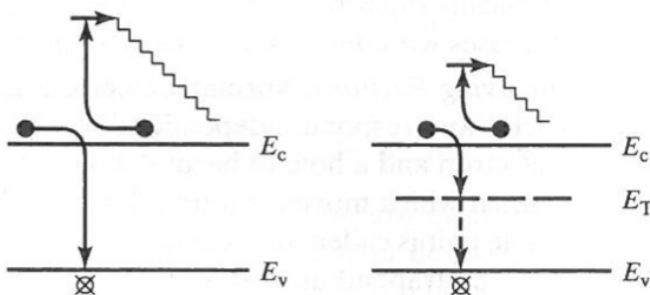
(b) R-G center recombination



(c) Recombination via shallow levels



(d) Recombination involving excitons



(e) Auger recombination

(a) Band-to-band recombination:

Electron-hole pair generated by an absorption event recombines directly emitting a photon. Electron returns to the valence band and “fills” the hole.

(b) R-G center recombination.

If there are energy levels (states) inside the bandgap. These levels can act as recombination-generation (R-G) centers. Electrons can jump down into these lower energy states releasing excess energy as heat (lattice vibrations = phonons).

(c) Recombination via shallow levels.

If there are exist energy levels inside the bandgap close to the conduction or valence band electrons or holes can jump onto these levels and then recombine with each other releasing the electron energy as a photon. (This process is something in between (a) and (b)).

(d) Recombination involving excitons.

This recombination process is analogous to band-to-band recombination except here electron and hole first form a bound pair – exciton with the energy slightly lower than that of the bandgap. Excitons behave similarly to a hydrogen atom, where hole acts as a nucleus with electron bound to it.

(e) Auger recombination.

This mechanism involves an additional electron. Here energy released during e-h recombination is used to promote the other electron onto higher energy level, from which it then falls down releasing energy as heat.

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3.024 Electronic, Optical and Magnetic Properties of Materials
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