Philosophy 244: #16— Contingent Identity

Dispensability

Names have often been seen as linguistic outliers and philosophically problematic. The more verbs and predicates you know, the larger your vocabulary, but it doesn't seem to increase your vocabulary to know a lot of names. They seem on the one hand descriptive—how otherwise would you find the referent? how otherwise could an identity statement involving names be informative?–and on the other hand not— Godel could have turned out not to be the author of "Godel's Theorem," but he could not have turned out not to be Godel.

These problems would be avoided, or eased, if names were theoretically dispensable, and would not occur in a logically perfect language. Quine argued that this indeed the case. For each would-be name a, he would have us introduce a one-place predicate φ_a such that $<o,w>\epsilon V(\varphi)$ iff $o=V(a)$. The role of a would then be played by $(\iota x: \varphi_a x)$. Instead of $\psi(a)$ we'd say $(\iota x{:}\varphi_a x)\psi(x)$. Russell's Theory of Descriptions can now be used to eliminate even the iota-phrase.

Here's a problem that may have occurred to you. Names lack scope. $\Box\psi(a)$ is unambiguous. Definite descriptions having scope, how can a description hope to take the place of a name? Suppose we want to necessitate $(\iota x{:}\varphi_a x)\psi(x)$. Is the result to be $\Box(\iota x{:}\varphi_a x)\psi(x)$ or $(\iota x{:}\varphi_a x)\Box\psi(x)$? As you might expect and hope, it turns out to make no difference; the truth value is the same in either case.

This should remind us of Russell's result about negation. Scope differences vis a vis negation collapse if the definite description is uniquely satisfied. Scope differences vis a vis necessitation collapse if the definite description is uniquely satisfied *in every world, and by the same object*. (This is rigidity.) Given that "nominal" definite descriptions are not subject to scope ambiguities, there is no danger of confusion if we write $(\iota x \varphi_a x)$ wherever we would have written a.

What about function symbols? An example of a binary function symbol is $+$ in arithmetic. It takes two terms t and t' and forms with them an additional term, viz. $t{+}t'$. Generally speaking if θ is an n-place function symbol and $t_1...$ are terms, then $\theta(t_1...t_n)$ is a term. V maps θ to a function $V(\theta)$ from n-tuples of domain elements to domain elements. The evaluation rule is

$[V\theta] \quad V_\mu(\theta(t_1....t_n))=V_\mu(\theta)(V_\mu(t_1)...V_\mu(t_n)).$

Functional terms are dispensable too, it turns out! The work of θ could be done by an n+1 place predicate φ_θ, where $\varphi_\theta(x_1....x_n y))$ means that $y = $ the result of applying θ to $x_1...x_n$. The role of $\theta(t_1....t_n)$ would be played by $(\iota y{:} \varphi_\theta(t_1....t_n y))$.

Note the absence of any world-variable in $[V\theta]$. Functional terms, like names, pick out the same thing in every possible world; they are *rigid*. Similarly it is always the same value of y that makes $V_\mu(\varphi_\theta(x_1....x_n y),w)=1$. Hence again the truth-value of $\Box(\iota y{:}\varphi_\theta(t_1....t_n y)) \psi(y)$ always agrees with that of $(\iota y{:}\varphi_\theta(t_1....t_n y)) \Box\psi(y)$. Functional definite descriptions having no scope-indicator role to play, we can write $(\iota y{:} \varphi_\theta(t_1....t_n y)$ wherever we would have written $\theta(t_1...t_n)$.

What's an example of a ternary function symbol? Can it be decomposed into binaries?

Non-Rigidity

Don't rejoice yet, however. All of this is because functional terms have been interpreted as rigid. A lot of English expressions with a functional look to them not are functional no longer. Take *oldest-child*(Pat) or *combined-weight*(Jupiter, Mars). These yield a different value in different worlds for a fixed input.

If we want to accommodate English functional terms, we need to be looking at non-rigid, world-relative function symbols. While we're at it we may as well look into the possibility of non-rigid *names*. Otherwise we will have to regard it as a necessary truth that Hesperus = Phosphorus. A necessary truth holds no matter what. How can a truth holding no matter what require empirical investigation to establish?

If we allowed names (or indeed variables) not to be rigid, there can be the same "contingent identity" phenomenon with them as we saw with definite descriptions:

the *F* = the *G*, but it could have been that the *F* ≠ the *G*.
a = *b*, but it could have been that *a*≠*b*.
x = *y*, but it could have been that *x*≠*y* (for some *x* and *y*)

All of these look at first sight like counterexamples to the classical view of identity, which validates as we saw □I: $\forall x \forall y \ (x=y \supset \Box x=y)$. The classical view can be defended against "counterexamples" of the first sort, by means of scope ambiguity. But there are also prima facie counterexamples involving names (*Hesperus=Phosphorus*, *Superman=Clark Kent* and variables (If *x = Hesperus* and *y = Phosphorus* , then *x=y* but $\Diamond(x \neq y)$, and these, it seems, don't *have* scope. How then can we hope to arrange for contingent identities involving names or variables?

Implementation

Since □I is a consequence of the fact that V(=) includes only ordered triples <o,o',w> such that o=o', our first thought might be to broaden the interpretation of = so that <o,o',w> with o≠o' are allowed into V(=). This response however would completely invalidate the Leibniz axiom: $x=y \supset (\alpha[x] \supset \alpha[y])$. And it would invalidate it even for non-modal instances of α; you could have identicals one of which is, say, an even number and the other of which is odd. Not good.

Messing with the interpretation of = is not the solution. What is? Look again at *The first PMG = the IB*. If this is to be an example of a contingent identity, what we want is that there's a person Ben in the actual world who's both the first PMG and the IB, and two people Benjamin and Benjie in another world, one of whom is the first PMG and the other of whom invented bifocals. Let's try to contsruct a counterexample to □I (*x=y⊃□x=y*) on that model. In the actual world, *x* and *y* are assigned the same object, say Ben. In some other worlds, *x* is assigned Benjamin and *y* is assigned Benjie. Ah hah, but then variable assignments are going to have to become world-relative; instead of $\mu(x)$ we'll have $\mu(x,w)$, and the rule for evaluating atomic wffs will be

[Vφ'] $V_{\mu}(\varphi(x_1....x_n),w)=1$ iff $<\mu(x_1,w)....,\mu(x_n,w),w> \epsilon V(\varphi)$

All the other rules remain as before except that in [V∀], we need to generalize the notion of an *x*-alternative: ρ is an *x*-alternative of μ iff for every variable *y* except (maybe) *x*, and every wϵW, $\rho(y,w)=\mu(y,w)$.

This semantics does not verify □I, because $\mu(x,w) = \mu(y,w) \not\Rightarrow \mu(x,u) = \mu(y,u)$. As a result it doesn't verify all instances of I2, Leibniz's axiom. On the other hand it *does* validate all instances of I2 in which α and β contain no modal operators. Which is maybe all we can expect if we're allowing contingent identity; if *x* just happens to be *y*, then things may be possible for *x* that are not possible for *y*, e.g. being distinct from *y*.

Converse Carban

This is certainly something to try, but a funny result should be noticed right away. Talking about interactions between quantifiers and modal operators, we noticed that three of the following four had at least some plausibility, while the fourth was riiculous:

Barcan Formula	$\forall x\Box\alpha\supset\Box\forall x\alpha$
Converse Barcan	$\Box\forall x\alpha\supset\forall x\Box\alpha$
Carban Formula	$\exists x\Box\alpha\supset\Box\exists x\alpha$
Converse Carban	$\Box\exists x\alpha\supset\exists x\Box\alpha$

The present semantics however validates this fourth formula. Why? Let <WRDV> be any model, w any world, and μ any (world-relative) variable assignment. If $V_\mu(\Box\exists x\alpha,w)=1$, then every world u that w can see has $V_\mu(\exists x\alpha,u)=1$; which means that for every such u, there is an *x*-alternative ρ^u to μ such that $V_{\rho^u}(\alpha,u)=1$

Now the interesting part. Suppose we gather together the objects o^u that ρ^u assigns to *x* as u varies. If $\rho(u)=o^u$ for each of these worlds, then clearly $V_\rho(\alpha,u)=1$ for each u that w can see. But then $V_\rho(\Box\alpha,w)=1$, whence $V_\mu(\exists x\Box\alpha,w)=1$. So Converse Carban, which seemed a moment ago ridiculous, comes out valid.

Why absurd? In competitive games there has to be a winner, but there is no one of whom it is necessary that *she* will win. It has to be that some card is on top, but there is no particular card which has to be the one on top. Always Zsa Zsa's husband is young, but none osf these husbands is always young.

The only way to make this plausible is to think of *the winner* as a peculiar sort of role-object, an object which is me in worlds where I win and you in worlds where you do. One can say in these terms that *The winner tends to think it was skill*, and even invoke that individual anaphorically—*and gets sniffy if you tell him it was just luck*. The temporal analogue would be to link past and present winners together into a single quasi-person. This is unnatural in many cases but in others less so, e.g., in a discussion of democratic ideals we may say that *The president has become less powerful than the Chair of the Federal Reserve*

Intensional Objects

Entities obtained by stringing together "ordinary" entities, one per world and/or time, are called *intensional objects*. They give us a new and more charitable take on Con-

Carnap in *Meaning and Necessity*, one of the first book-length treatments of quantified modal logic, accepted this reasoning, and its supposedly ridiculous upshot.

verse Carban. If there has to be someone in charge of the others, there's an intensional entity *The Boss* (or *Mr Big*) who is necessarily in charge of the others. If the first bird on a given morning is the only one who can be sure to get a worm, then there's an elusive further character *the early bird* who always gets *the worm*.

Here's the problem: on the intensional object interpretation, which is what we're trying now, Converse Carban comes out valid. But it shouldn't if we're interested in contingent identity systems, for these are supposed to *weaker* than earlier systems, and Converse Carban wasn't a theorem of those earlier systems.

This kind of trouble arises because we're allowing intensional objects to be introduced at will. Suppose we try to limit the construction somehow. An intensional object is a function $\mathcal{I}$ from worlds into the domain. A model as a quintuple $<$WRDIV$>$ where I is a set of intensional objects. Variable assignments μ will now take variables not to domain elements but members of this set. Instead of $\mu(x,w)$ we now have $\mu(x) = \mathcal{I}$, a function from worlds to regular objects. An x-alternative to μ is a ρ taking every y distinct from x to the same intensional object as μ; $\rho(y)=\mu(y)$.

[Vφ"] $V_\mu(\varphi(x_1....x_n),w)=1$ iff $<\mu(x_1)(w)....\mu(x_n)(w),w>\epsilon V(\varphi)$

[V$\forall$] $V_\mu(\forall x\alpha,w)=1$ iff $V_\rho(\alpha,w)=1$ for every x-alternative ρ of μ

Notice something funny here. When we're interpreting *predicates*, variables contribute regular "extensional" objects, members of the domain: $\mu(x)(w) = \mathcal{I}(w) = $ o. When we're working with *quantifiers*, they range over intensional objects, or functions from worlds to domain-elements.

Logical Systems

How is contingent identity to be dealt with axiomatically? The Leibniz axiom has got to be limited so that α is non-modal; in fact it suffices to apply it just to predicates:

I2" $x=y\supset(\varphi(x)\supset \varphi(y))$.

S+BF with the addition of I1 and I2" will be S+CI. Soundness is easy. For completeness we again need a canonical model. Everything is as before except we have to say what will play the role of I, the set of intensional objects. For each w, let $\mathcal{I}_x(x)(w)$ = the first y such that $x=y\epsilon$w. I is the set of $\mathcal{I}_x$ for each variable x, and the canonical assignment μ takes each x to $\mathcal{I}_x$. As usual, $<x_1....x_n,w>\epsilon V(\varphi)$ iff $\varphi(x_1...x_n)\epsilon$w.

Prop. 18.1 $V_\mu(\alpha,w)=1$ iff $\alpha\epsilon$w.

Completeness follows as before. If α is valid then it holds in all worlds of all models built on suitable frames. If the canonical model is built on the right sort of frame, then α holds in all worlds of the canonical model, that is, in all maximal consistent sets. A statement belonging to all maximal consistent sets is a theorem. Hence every valid α is a theorem, as promised.

There's some real flexibility in the amount of contingent identity (distinctness) we choose to allow. Say we want a semantics validating □I ($x=y\supset\Box x=y$) but not necessarily □NI ($x \neq y\supset\Box x \neq y$). Then we stipulate that when wRu, if $\mathcal{I}_1$ and $\mathcal{I}_2$ agree on w, they agree on u. While not stipulating that when wRu, if $\mathcal{I}_1$ and $\mathcal{I}_2$ agree on u, they agree on w. NEXT WEEK: quantifying over all intensional objects, and why the results are not axiomatizable.

A good portion of Carnap's *Meaning & Necessity* is devoted to defending this double-dealing, which he called "The Method of Extension and Intension," and Quine called "Carnap's curious double interpretation of the variables."

24.244 Modal Logic
Spring 2015