Logic I
Fall 2009
Session 18 Handout

Formal semantics for PL

- An *interpretation* for PL is a specification of each of the following:
  - A universe of discourse UD, where $\emptyset \subset$ UD.
  - For each sentence letter **S** of SL, a truth-value assigned to **S**.
  - For each n-place predicate letter **F** of PL, a set of n-tuples of members of UD assigned to **F**.

    E.g., if we are interpreting 'Lxy' to mean *x lives in y*, then to 'L' we would assign $\{ \langle$ Ephraim, Somerville $\rangle$, $\langle$ Damien, Somerville $\rangle$, $\langle$ Vann McGee, Boston $\rangle$, $\ldots \}$.
  - For each individual constant **c** of PL, an individual u assigned to **c**, where u is a member of UD.

- *Denotations* of individual terms on **I** and **d**:
  - If **t** is a variable, then $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}) = \mathbf{d(t)}$.
  - If **t** is an individual constant, then $\text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}) = \mathbf{I(t)}$.

- Variants of variable assignments:
  - **d**[u/**x**] is the variable assignment that assigns u to **x** and is otherwise just like **d**.

  E.g., if $\text{d}_1 = \{\langle$ Alice, **x** $\rangle$, $\langle$ Bill, **y** $\rangle$, $\langle$ Carol, **z** $\rangle$, $\ldots\}$, then we have:
  d_1[John/**y**] $= \{\langle$ Alice, **x** $\rangle$, $\langle$ John, **y** $\rangle$, $\langle$ Carol, **z** $\rangle$, $\ldots\}$

- *Satisfaction* for formulas of PL:
  1. If **P** is a sentence letter, **d** satisfies **P** on **I** iff **I**(**P**)=T.
  2. If **P** is an atomic formula of the form $\mathbf{At}_1 \ldots \mathbf{t}_n$, where **A** is an n-place predicate of PL and $\mathbf{t}_1 \ldots \mathbf{t}_n$ are individual terms of PL, **d** satisfies **P** on **I** iff $\langle\ \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_1), \ldots, \text{den}_{\mathbf{I},\mathbf{d}}(\mathbf{t}_n)\ \rangle \in \mathbf{I(A)}$.
  3. If **P** is of the form $\neg$**Q**, then **d** satisfies **P** on **I** iff **d** does not satisfy **Q** on **I**.
  4. If **P** is of the form **Q**&**R**, then **d** satisfies **P** on **I** iff **d** satisfies **Q** on **I** or **d** satisfies **R** on **I**.
  5. If **P** is of the form $\mathbf{Q} \vee \mathbf{R}$, then **d** satisfies **P** on **I** iffeither **d** satisfies **Q** on **I** or **d** satisfies **R** on **I**.

6. If $\mathbf{P}$ is of the form $\mathbf{Q} \supset \mathbf{R}$, then $\mathbf{d}$ satisfies $\mathbf{P}$ on $\mathbf{I}$ iffeither $\mathbf{d}$ doesn't satisfy $\mathbf{Q}$ on $\mathbf{I}$ or $\mathbf{d}$ satisfies $\mathbf{R}$ on $\mathbf{I}$.
7. If $\mathbf{P}$ is of the form $\mathbf{Q} \equiv \mathbf{R}$, then $\mathbf{d}$ satisfies $\mathbf{P}$ on $\mathbf{I}$ iffeither $\mathbf{P}$ and $\mathbf{Q}$ are both satisfied by $\mathbf{d}$ on $\mathbf{I}$ or neither $\mathbf{P}$ nor $\mathbf{Q}$ is satisfied by $\mathbf{d}$ on $\mathbf{I}$.
8. If $\mathbf{P}$ is of the form $(\forall \mathbf{x})\mathbf{Q}$, then $\mathbf{d}$ satisfies $\mathbf{P}$ on $\mathbf{I}$ iff for any u $\in$ UD, d[u/$\mathbf{x}$] satisfies $\mathbf{Q}$ on $\mathbf{I}$.
9. If $\mathbf{P}$ is of the form $(\exists \mathbf{x})\mathbf{Q}$, then $\mathbf{d}$ satisfies $\mathbf{P}$ on $\mathbf{I}$ iff for some u $\in$ UD, d[u/$\mathbf{x}$] satisfies $\mathbf{Q}$ on $\mathbf{I}$.

- Truth for PL sentences:
  - A sentence $\mathbf{P}$ is true on interpretation $\mathbf{I}$ iff every variable assignment $\mathbf{d}$ for I[1] satisfies $\mathbf{P}$ on $\mathbf{I}$.

Formal semantics for PLE

- The definition of truth is the same.
- We extend the definitions of satisfaction, denotation, and interpretation.
- An interpretation for PLE includes all elements of an interpretation for PL plus:
  - An assignment of a set of n+1-tuples to each n-place functor of PLE.

  E.g., if we want to interpret 'g(x,y)' to mean *the sum of x and y*, then to 'g' we would assign $\{ \langle 1,1,2 \rangle, \langle 1,2,3 \rangle, \langle 1,3,4 \rangle \ldots \}$.

- Denotations in PLE are the same with the addition of a clause for functor-terms:
  - If $\mathbf{t}$ is a term $\mathbf{f}(\mathbf{t_1}, \ldots, \mathbf{t_n})$, where $\mathbf{f}$ is an n-place functor, $\mathbf{t_1}, \ldots, \mathbf{t_n}$ are terms, and $\langle$ den$_{\mathbf{I,d}}(\mathbf{t}_1)$, ..., den$_{\mathbf{I,d}}(\mathbf{t}_n)$, u $\rangle \in \mathbf{I(f)}$, then den$_{\mathbf{I,d}}(\mathbf{t})$= u.

  E.g., in the above example, if we let our term t be 'g(1,3)', then since 'g' is a two-place functor and $\langle$ 1,3,4 $\rangle \in$ g, den$_{\mathbf{I,d}}(\mathbf{t})$= 4.

- Clause (2) of the definition of *satisfaction* already covers functor-expressions, since these count as individual terms. But we need to add a clause for '=':

10. If $\mathbf{P}$ is an atomic formula of the form $\mathbf{t_1} = \mathbf{t_2}$, then $\mathbf{d}$ satisfies $\mathbf{P}$ on interpretation $\mathbf{I}$ iffden $_{\mathbf{I,d}}(\mathbf{t_1})$ = den$_{\mathbf{I,d}}(\mathbf{t_2})$

[1] Variable assignment d is *for I* iff d assigns only objects in I's UD.

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