All the interpretations I provide are relatively informal. It's fine if you did things the informal way I did them. It's fine if you did things more formally too. Future problem sets may require you to provide formal interpretations, however.

I have explained why sentences are true or false on given interpretations in different ways for different answers, in the hope that at least one of these ways will make sense to you. All the different ways of explaining things are equivalent.

7.8E

Question 2

Part (d)

$$(\forall x)((Ax \mathbin{\&} (\forall y)(Fy \supset Exy)) \supset Rx)$$

Part (k)

$$(\exists x)(Fx \mathbin{\&} (\forall y)(Ay \supset Dxy)) \mathbin{\&} (\exists x)(Ax \mathbin{\&} (\forall y)(Fy \supset Dxy))$$

Part (l)

$$(\exists x)((Ax \mathbin{\&} Fx) \mathbin{\&} (\forall y)((Ay \mathbin{\&} \sim Fy) \supset Dxy))$$

8.1E

Question 1

Part (d)

'$Cb \equiv (\sim Ab \equiv Ac)$' is false on the given interpretation. -4 is not a positive number, so 'Ac' is false on the given interpretation, and 39 is a positive number, so 'Ab is true, and, hence '$\sim Ab$' is false on the given interpretation. So the sentence on the right of the main '$\equiv$' is true on the given interpretation. And, as 39 is not a negative number, 'Cb' is false on the given interpretation. So the whole biconditional is false on the given interpretation.

Part (f)

'$\sim (\sim Ab \vee Cb) \supset Baa$' is true on the given interpretation. A quick way to see this: 0 is its own square root, so 'Baa' is true on the given interpretation. So the right-hand side of the conditional is true on the given interpretation. So the whole conditional is true on the given interpretation.

Part (h)

$\sim(Ab \vee Bcc) \ \& \ (Cc \supset \sim Ac)$ is false on the given interpretation. A quick way to see this: 39 is a positive number, so 'Ab' is true on the given interpretation. So the disjunction inside the left part of the sentence is true. So the negation of that disjunction — i.e., the whole of the left-hand of the sentence — is false. So, as the sentence is a conjunction, the sentence as whole is false.

8.2E

Question 2

Part (d)

An arbitrary example:
UD: {Damien}
Fx: x likes peanut butter.

'$(\exists x)((\exists y)Fy \supset \sim Fx)$' is true on this interpretation. Why? Well, I don't like peanut butter, so there's nothing in the universe of discourse that likes peanut butter, so the antecedent of the embedded conditional is false, on this interpretation. So plugging my name in for 'x' satisfies the embedded conditional. So the sentence is true on this interpretation.

Question 3

Part (a)

An interpretation on which '$(\exists x)(Fx \ \& \ Gx) \supset (\exists x)\sim(Fx \vee Gx)$' is true:
UD: {Damien}
Fx: x likes peanut butter.
Gx: x was bitten by a monkey.

I don't like peanut butter, so there is no way of assigning objects in the universe of discourse to variables that satisfies '$(Fx \ \& \ Gx)$', so the antecedent of the conditional is false, so the whole conditional is true, on this interpretation.

An interpretation on which '$(\exists x)(Fx \ \& \ Gx) \supset (\exists x)\sim(Fx \vee Gx)$' is false:
UD: {Damien}
Fx: x likes pancakes
Gx x was bitten by a monkey.

I both like pancakes and was bitten by a monkey, so there is something in the universe of discourse that both likes pancakes and was bitten by a monkey, so the antecedent of the conditional is true. And, seeing as I like pancakes, either I like pancakes or I was bitten by a monkey, so I am not such that it is not case that either I like pancakes or I was bitten by a monkey, so there is nothing in the universe of discourse such that it is not the case that either it likes pancakes or it was bitten by a monkey, so the consequent of the conditional is false. So the antecedent of the conditional is true and the consequent is false. So the conditional is false, on this interpretation.

8.3E

Question 1

Part (h)

UD: {Damien, Ephraim}
Mx: x is a TA.
Ny: y has a PhD.

I am a TA, and I don't have a PhD. Ephraim has a PhD, and is not a TA. So nothing in the universe of discourse is such that it is a TA iff it has a PhD. So the'$(\exists y)(My \equiv Ny)$' is false, on this interpretation.

But there is something that is a TA — namely, me. And there is something that has a PhD — namely, Ephraim. So both sides of the biconditional '$(\exists y)My \equiv (\exists y)Ny$' are true, on this interpretation. So '$(\exists y)My \equiv (\exists y)Ny$' is true, on this interpretation.

Question 4

Part (f)

UD: {Damien, Ephraim}
Fx: x wears glasses.
Bxy: x is shorter than y

Ephraim wears glasses, so there is something in the universe of discourse that wears glasses, so '$(\exists w)Fw$' is true, on this interpretation.

I don't wear glasses, so the conditional embedded in '$(\forall w)(Fw \supset (\exists x)Bxw)$' is true when you plug in my name for 'w' (as the antecedent of the conditional is false). And there is something in the universe of discourse shorter than Ephraim — namely, me — so the embedded conditional is true when you plug in Ephraim's name for 'w' (as the consequent of the conditional is true). And we're the only things in the universe of discourse. So '$(\forall w)(Fw \supset (\exists x)Bxw)$' is true, on this interpretation.

Neither Ephraim nor I are shorter than ourselves, and, again, we're the only things in the universe of discourse, so '$(\forall x) \sim Bxx$' is true, on this interpretation.

8.4E

Question 2

Part (f)

UD: {Damien, Ephraim}
    Mxy: x has bigger feet than y
    Nxy: x is taller than y.

'$(\forall x)(\forall y)(Mxy \supset Nxy)$' is true on this interpretation, as the only pair of objects in the UD that satisfies the antecedent of the embedded conditional is $\langle$Ephraim, Damien$\rangle$ (i.e., Ephraim has bigger feet than I, but I don't have bigger feet than he, and neither of us have bigger feet than ourselves), and that pair also satisfies that consequent (as Ephraim is taller than I am).

'$(\forall x)(\forall y)(Mxy \supset (Nxy \ \& \ Nyx))$' is false on this interpretation, as it is not the case that for every pair that satisfies the antecedent of the embedded conditional, that pair satisfies the consequent. The only pair that satisfies the antecedent is $\langle$Ephraim, Damien$\rangle$, as I said, but it is not the case that I am taller than Ephraim, so it is not the case that Ephraim is taller than I am and I am taller than Ephraim is, so the consequent is not satisfied by that pair.

So every member of the premise set is true on the this interpretation, and the conclusion is false on this interpretation, So this argument is quantificationally invalid.

Part (g)

UD: {Damien, Ephraim}
    Gx: x has a pet mouse.
    Dxy: x has the same birthday as y

I have a pet mouse, so '$(\exists x)Gx$' is true, on this interpretation.

I have the same birthday as myself, and Ephraim has the same birthday as himself, so '$(\forall x)(Gx \supset Dxx)$' is true on this interpretation.

I do not have the same birthday as Ephraim and he does not have the same birthday as me, so neither of us are such that we have the same birthday as everything in the UD. So neither of us are such that we have a pet mouse and we have the same birthday as everything in the UD. So '$(\exists x)(\forall y)(Gx \ \& \ Dxy)$' is false, on this interpretation.

So every member of the premise set is true on the this interpretation, and the conclusion is false on this interpretation, So this argument is quantificationally invalid.