

Question 1

Of course, there are a lot of examples. Here are the first ones that came to my mind that weren't completely uninteresting.

Part (a)

Here's one:

$$(\forall x)(Rxx \supset (\exists y)Rxy)$$

(In quasi-English: 'For all things x , if x bears relation R to itself, then there is something to which x bears relation R .')

Part (b)

The negation of the sentence above will do. But for variety's sake, here another example:

$$(\exists x)(Fx \& \sim Fx)$$

(In quasi-English: 'There exists something such that it is F and it is not the case that it is F .')

This one is pretty uninteresting, I admit.

Part (c)

This pair is kind of interesting:

$$(\forall x)(Fx \supset (A \& \sim A))$$

$$\sim (\exists x)Fx$$

(In quasi-English: 'For all x , if x is F , then a contradiction is true.' 'Nothing is F .')

Part (d)

Voilà:

$$Fa$$

$$\sim (\exists x)Fx$$

(In quasi-English: ' a is F .', 'Nothing is F .')

Not something you need to know, but for those who are interested: this pair is quantificationally incompatible in PL. However, there are alternative languages such that sentences like these are *not* incompatible. The derivation systems that go with those languages are called 'free logics'.

Part(e)

Check it out:

$$\{(\forall x)(\forall y)(Rxy \supset Ryx), (\forall x)(\forall y)(\forall z)((Rxy \& Ryz) \supset Rxz)\}$$
$$(\forall x)Rxx$$

(In quasi-English: {'For all x and for all y , if x bears R to y , then y bears R to x .'}, 'For all x , y and z , if x bears R to y and y bears R to z , then x bears R to z .'}, and lastly 'For all x , x bears R to x .')

Again, just for the interested: a relation R is what is called 'symmetric' iff the first sentence in the set above is true of it. R is what is called 'transitive' iff the second sentence in the set above is true of it. The last sentence, outside the set, is true iff R is 'reflexive'. What I am claiming, in claiming that the above set entails the above sentence, is that every symmetric, transitive relation is reflexive. And that's true.

Question 2

Here is such a sentence:

$$(\forall x)(Fx \supset Gx)$$

There is no equivalent sentence of the other form because the only candidate is the string of symbols $(\exists x)Fx \supset Gx$, and that is not a sentence of PL — the third ' x ' is an unbound variable, and no sentence of PL contains an unbound variable.

Question 3 (7.8E, part 2)

There is more than one way to do each of these. I only provide one answer to each question here, but if your answer is something else that is obviously equivalent, that's ok. (If your answer is equivalent, but not obviously so, that could be bad.)

(c)

$$\sim (\exists x)(Fx \& Ex)$$

The main logical operator is the ' \sim '.

(h)

$$(\forall x)((Fx \& Ax) \supset (\exists y)(Exy \& (Fy \& \sim Ay)))$$

The main logical operator is the ' $(\forall x)$ '.

(i)

$$(\exists x)(Ax \& \sim Fx \& (\forall y)((Fy \& Ay) \supset Exy))$$

The main logical operator is the ' $\exists x$ '.

(n)

$$(\forall x)(\exists y)Dxy \& (\forall x)(\forall y)(Dxy \supset (Ax \& \sim Fx))$$

The main logical operator is the '&'.

Question 4 (7.8E, part 5)

Even more ways to do these. Again, as long as your answer clearly means that same thing as what I have written, and it is fluent English, you're ok.

I use 'number' here to mean 'positive integer' (as it does, in many contexts).

(a)

The product of an even number and any number is an even number.

(h)

There is an even prime number.

(n)

If one number is larger than another, the second number is not larger than the first.

(r)

There is an even prime number, and every prime number larger than it is odd.

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