Russell's Theory of Definite Descriptions

"The person who wrote *Waverley* wrote *Ivanhoe*" logically implies "There is someone who wrote both *Waverley* and *Ivanhoe*." Bertrand Russell[1] figured out a clever way to formalize the inference, using machinery we already have on hand. Up until now, we'd have to symbolize "the person who wrote *Waverley*" by an individual constant, so that our inference would take the form:

$$\begin{array}{l} Ia \\ \therefore\ (\exists x)(Wx \wedge Ix) \end{array}$$

which is obviously invalid. (Let us restrict our universe of discourse to human beings, so we don't have to symbolize "person.") Russell introduced a new symbolism for expressions consisting of "the" + a singular common noun. He symbolized "the person who wrote *Waverley*" as "$(\iota x)Wx$," and he symbolized "The person who wrote *Waverley* wrote *Ivanhoe*" as "$I(\iota x)Wx$."

According to Russell, "The person who wrote *Waverley* wrote *Ivanhoe*" is logically equivalent to the conjunction of these three statements:

At least one person wrote *Waverley.*
At most one person wrote *Waverley.*
Anyone who wrote *Waverley* wrote *Ivanhoe*.

In symbols,

$$(((\exists x)Wx \wedge \neg(\exists x)(\exists y)(\neg x{=}y \wedge (Wx \wedge Wy))) \wedge (\forall x)(Wx \rightarrow Ix)).$$

Equivalently, and more simply,

$$(\exists x)((\forall y)(Wy \leftrightarrow y{=}x) \wedge Ix).$$

Russell proposed to use this last sentence to symbolize "The person who wrote *Waverley* wrote *Ivanhoe*."

In general, a sentence of the form "The F is a G" is symbolized "$G(\iota x)Fx$," which is regarded as an abbreviation of "$(\exists x)((\forall y)(Fx \leftrightarrow y{=}x) \wedge Gx)$."

"The person who wrote *Waverley* exists" is, according to Russell, equivalent to the conjunction of two sentences:

At least one person wrote *Waverley.*
At most one person wrote *Waverley.*

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[1] "On Denoting," *Mind* n.s. 14 (1905): 479-93.

In symbols,

$$(\exists x)Wx$$
$$\neg(\exists x)(\exists y)(\neg x{=}y \wedge (Wx \wedge Wy))$$

which can be written more simply thus:

$$(\exists x)(\forall y)(Wy \leftrightarrow y{=}x).$$

In general, Russell proposed to symbolize sentences of the form “The F exists” as “$E!(\iota x)Fx$,” which he treated as an abbreviation for “$(\exists x)(\forall y)(Fy \leftrightarrow y{=}x)$.”

“The present king of France is bald” — in symbols, “$B(\iota x)Kx$” — is false, says Russell, because it entails “The present king of France exists” — “$E!(\iota x)Kx$” — and there is, at present, no king of France. We can prove this by doing a derivation:

1	1 $B(\iota x)Kx$	PI
1	2 $(\exists x)((\forall y)(Ky \leftrightarrow y{=}x) \wedge Bx)$	Unabbreviated form of 1
3	3 $((\forall y)(Ky \leftrightarrow y{=}a) \wedge Ba)$	PI (for ES)
3	4 $(\forall y)(Ky \leftrightarrow y{=}a)$	TC, 3
3	5 $(\exists x)(\forall y)(Ky \leftrightarrow y{=}x)$	EG, 4
1	6 $(\exists x)(\forall y)(Ky \leftrightarrow y{=}x)$	ES, 2, 3, 5
1	7 $E!(\iota x)Kx$	Abbreviated form of 6
	8 $(B(\iota x)Kx \rightarrow E!(\iota x)Kx)$	CP, 1, 7

Our original inference about *Waverley* and *Ivanhoe* is symbolized as follows:

$$I(\iota x)Wx$$
$$\therefore\ (\exists x)(Wx \wedge Ix)$$

1	1 $I(\iota x)Wx$	PI
1	2 $(\exists x)((\forall y)(Wy \leftrightarrow y{=}x) \wedge Ix)$	Unabbreviated form of 1
3	3 $((\forall y)(Wy \leftrightarrow y{=}a) \wedge Ia)$	PI (for ES)
3	4 $(\forall y)(Wy \leftrightarrow y{=}a)$	TC, 3
3	5 Ia	TC, 3
3	6 $(Wa \leftrightarrow a{=}a)$	UG, 4
	7 $a{=}a$	IR
3	8 Wa	TC, 6, 7
3	9 $(Wa \wedge Ia)$	TC, 5, 8
3	10 $(\exists x)(Wx \wedge Ix)$	EG, 9
1	11 $(\exists x)(Wx \wedge Ix)$	ES, 2, 3, 10

We symbolize

Scott is the author of *Waverley*.
The author of *Waverley* is a poet.
Therefore, Scott is a poet.

as follows:

$s = (\iota x)Wx$
$P(\iota x)Wx$
$\therefore\ Ps$

1	1 $s = (\iota x)Wx$	PI
2	2 $P(\iota x)Wx$	PI
1	3 $(\exists x)((\forall y)(Wy \leftrightarrow y=x) \wedge s=x)$	Unabbreviated form of 1
2	4 $(\exists x)((\forall y)(Wy \leftrightarrow y=x) \wedge Px)$	Unabbreviated form of 2
5	5 $((\forall y)(Wy \leftrightarrow y=a) \wedge s=a)$	PI (for ES)
5	6 $(\forall y)(Wy \leftrightarrow y=a)$	TC, 5
5	7 $s=a$	TC, 6
8	8 $((\forall y)(Wy \leftrightarrow y=b) \wedge Pb)$	PI (for ES)
8	9 $(\forall y)(Wy \leftrightarrow y=b)$	TC, 8
8	10 Pb	TC, 8
5	11 $(Ws \leftrightarrow s=a)$	US, 6
5	12 Ws	TC, 7, 11
8	13 $(Ws \leftrightarrow s=b)$	US, 9
5,8	14 $s=b$	TC, 12, 13
5,8	15 Ps	SI, 10, 14
2,5	16 Ps	ES, 4, 8, 15
1,2	17 Ps	ES, 3, 5, 16

Now let's do this inference:

The eldest son of a Greek hero, if he exists, is a hero.
Odysseus is a Greek hero.
Telemachus is the eldest son of Odysseus.
Therefore, Telemachus is a hero.

In symbols,

$(\forall x)((Gx \wedge Hx) \rightarrow (E!(\iota y)Syx \rightarrow H(\iota y)Syx))$
$(Go \wedge Ho)$
$t = (\iota x)Sxo$
$\therefore\ Ht$

1	1 $(\forall x)((Gx \wedge Hx) \rightarrow (E!(\iota y)Syx \rightarrow H(\iota y)Syx))$	PI
2	2 $(Go \wedge Ho)$	PI
3	3 $t = (\iota x)Sxo$	PI
1	4 $(\forall x)((Gx \wedge Hx) \rightarrow ((\exists y)(\forall z)(Szx \leftrightarrow z=y) \rightarrow (\exists y)((\forall z)(Szx \leftrightarrow z=y) \wedge Hy)))$	Unabbreviated form of 1
3	5 $(\exists x)((\forall y)(Syo \leftrightarrow y=x) \wedge t=x)$	Unabbreviated form of 3
1	6 $((Go \wedge Ho) \rightarrow ((\exists y)(\forall z)(Szo \leftrightarrow z=y) \rightarrow (\exists y)((\forall z)(Szo \leftrightarrow z=y) \wedge Hy)))$	US, 4
1,2	7 $((\exists y)(\forall z)(Szo \leftrightarrow z=y) \rightarrow (\exists y)((\forall z)(Szo \leftrightarrow z=y) \wedge Hy))$	TC, 2, 7
8	8 $((\forall y)Syo \leftrightarrow y=a) \wedge t=a)$	PI (for ES)
8	9 $(\forall y)(Syo \leftrightarrow y=a)$	TC, 8
8	10 $t=a$	TC, 8
8	11 $(\forall y)(Syo \leftrightarrow y=t)$	SI, 9, 10
8	12 $(Sbo \leftrightarrow b=t)$	US, 11
8	13 $(\forall z)(Szo \leftrightarrow z=t)$	UG, 12
8	14 $(\exists y)(\forall z)(Szo \leftrightarrow z=y)$	EG, 13
1,2,8	15 $(\exists y)((\forall z)(Szo \leftrightarrow z=y) \wedge Hy)$	TC, 7, 14
16	16 $((\forall z)(Szo \leftrightarrow z=c) \wedge Hc)$	PI (for ES)
16	17 $(\forall z)(Szo \leftrightarrow z=c)$	TC, 16
16	18 Hc	TC, 16
8	19 $(Sto \leftrightarrow t=t)$	US, 13
	20 $t=t$	IR
8	21 Sto	TC, 19, 20
16	22 $(Sto \leftrightarrow t=c)$	US, 17
8,16	23 $t=c$	TC, 21, 22
8,16	24 Ht	SI, 18, 23
1,2,8	25 Ht	ES, 15, 16, 24
1,2,3	26 Ht	ES, 5, 8, 25

There is an annoying ambiguity in the story we have told so far. Consider the sentence “The present king of France is not bald.” There are two ways we might symbolize the sentence. We might read it as the negation of the sentence obtained by substituting “the present King of France” for “x” in the open sentence “x is bald.” Alternatively, we might understand it as the result of substituting “the present king of France” for “x” in the open sentence “x is not bald.” The first choice gives us “$\neg(\exists x)((\forall y)(Ky \leftrightarrow y=x) \wedge Bx)$,” which is true, whereas the second gives us the false sentence, “$(\exists x)((\forall y)(Ky \leftrightarrow y=x) \wedge \neg Bx)$.”

Russell developed an unambiguous notation that made room for both readings. While admirably precise, it was rather cumbersome. The simplest policy would simply be always to cash out a definite description by applying Russell’s method to the atomic open sentence that contains the description. This policy will make out “The present king of France is not bald” to be true, which I would regard as intuitively correct. Under this policy, “The present king of France is either bald or not bald” comes out as a tautology, “$((\exists x)((\forall y)(Ky \leftrightarrow y=x) \wedge Bx) \vee$

$\neg(\exists x)((\forall y)(Ky \leftrightarrow y=x))$."

There's still an ambiguity. What do we do with an atomic sentence that contain several definite descriptions? And how do we cash out a definite description that contains another definite description inside it? These are small problems that we don't need to address here.