Subject 24.241. Logic I. Homework due Thursday, December 1

I. Please derive "$((\exists x)(Fx \wedge Gx) \rightarrow ((\exists x)Fx \wedge (\exists x)Gx))$" from the empty set.

II. Please derive "$((\forall x)(Fx \wedge Gx) \leftrightarrow ((\forall x)Fx \wedge (\forall x)Gx))$" from the empty set.

III. Please derive "$(\forall x)(Fx \leftrightarrow Fa)$" from {"$((\exists x)Fx \rightarrow (\forall x)Fx)$"}.

IV. Please derive "$((\exists x)Fx \rightarrow (\forall x)Fx$" from {"$(\forall x)(Fx \leftrightarrow Fa)$"}.

V. Please derive "$((\forall x)Fx \rightarrow (\exists x)Gx)$" from {"$(\exists x)(Fx \rightarrow Gx)$"}.

VI. Please derive "$(\exists x)(Fx \rightarrow Gx)$" from {"$((\forall x)Fx \rightarrow (\exists x)Gx)$"}

VII. Please translate the following argument, due to Lewis Carroll, into English, then derive the translated conclusion from the translated premisses:

Animals, that do not kick, are always unexcitable.
Donkeys have no horns.
A buffalo can always toss one over a gate.
No animals that kick are easy to swallow.
No hornless animal can toss one over a gate.
The only animals that aren't excitable are buffalos.
Donkeys and buffalos are animals.
Therefore, donkeys are never easy to swallow.

VIII. Rule ES tells us that, if we have derived $(\exists x)\varphi$ with premiss set Γ and we have derived ψ with premiss set $\Delta \cup \{\varphi^{x}/_{c}\}$, then we may derive ψ with premiss set $\Gamma \cup \Delta$, provided the following three caveats are met:

c doesn't occur within any of the members of Δ.
c doesn't appear in φ.
c doesn't appear in ψ.

Show, by giving three examples, that if any of the three caveats were omitted and the other rules remained the same, then there would be a derivation that derived an invalid sentence from the empty premiss set.

IX. True or false? Explain your answer. If all three caveats were removed from rule ES and the other rules all remained the same, then it would be possible to derive every sentence from the empty premiss set.