MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Departments of Electrical Engineering, Mechanical Engineering, and the Harvard-MIT Division of Health Sciences and Technology

6.022J/2.792J/BEH.371J/HST.542J: Quantitative Physiology: Organ Transport Systems

QUIZ 3

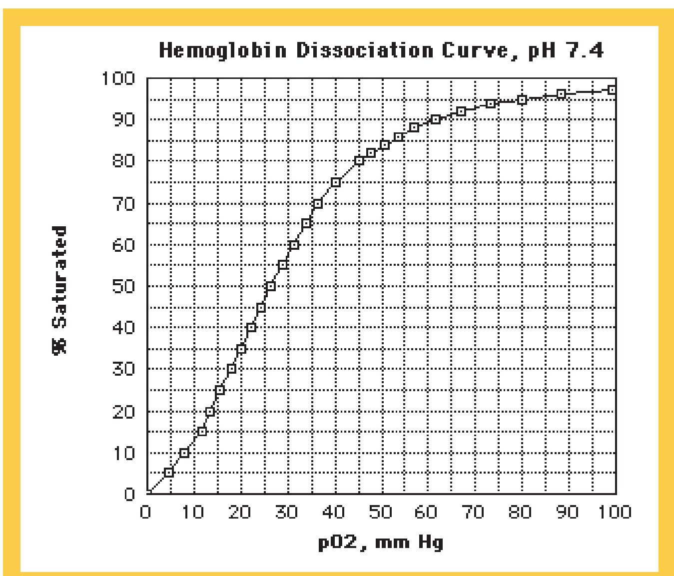
SOLUTIONS

These are normal values of physiological parameters for a 70 kg person.

R_{rs} (respiratory system R)	=	4	mbar·s/l
C_{cw}	=	200	ml/mbar
C_{lung}	=	200	ml/mbar
V_D (Anatomic)	=	150	ml
V'_{O_2}	=	274	ml/min
V'_{CO_2}	=	220	ml/min
RQ	=	0.8	
Q_s/Q_T (Shunt fraction)	$<$	0.05	
Q_T (cardiac output)	=	5	l/min
P_{atm}	=	760	mmHg
$P_{v_{CO_2}}$	=	46	mmHg
$P_{v_{O_2}}$	=	40	mmHg
$P_{a_{CO_2}}$	=	40	mmHg
$P_{a_{O_2}}$ (at room air)	=	100	mmHg
$(A-a)DO2$	$\approx$	6-10	mmHg
pH	=	7.4	
cHb	=	15	g/100ml-blood
Hb O_2 Binding capacity	=	20.1	ml O_2/100ml blood
FRC	=	2.4	l

The normal hemoglobin O_2 saturation curve is also included and should be used only when there is no alternative data available.

Figure 1:



The first two problems are cases that include certain respiratory physiologic abnormalities. You can use the normal values as a reference, or in absence of additional information.

Problem 1 (Case 1)

A patient comes to the emergency ward with shortness of breath and wheezing. He is breathing room air at a rate of 30 breaths per minute, and the pulse oximeter shows his arterial blood saturation to be $S_{a_{O_2}}$ = 0.80.

Arterial and mixed venous blood samples are taken at arrival and reveal the following values:

$$\begin{array}{lcrl} P_{v_{CO_2}} & = & 44 & \text{mmHg} \\ P_{v_{O_2}} & = & 27 & \text{mmHg} \\ P_{a_{CO_2}} & = & 39 & \text{mmHg} \\ P_{a_{O_2}} \text{ (at room air)} & = & 20 & \text{mmHg} \end{array}$$

The blood gas data comes with a computer generated caution questioning the validity of the measurements.

A. Please identify which of the four blood gas values may have an error and explain your reasoning. (25%)

$P_{a_{O_2}}$ cannot be correct for two reasons:

(i) $P_{a_{O_2}}$ *is always* $\geq P_{v_{O_2}}$

(ii) $P_{a_{O_2}} = 20$ *is not compatible with* $S_{a_{O_2}} = 0.8$*. In fact* $S_{a_{O_2}} \approx 0.35$.

B. You need to make a best guess to treat the patient with the knowledge available to you; can you find an approximate value of the erroneous blood gas? (25%)

For $S_{a_{O_2}} = 0.8$, $P_{a_{O_2}} = 45$ *(from the* O_2 *saturation curve).*

C. The patient is given 100% O_2 by mask and one hour later his blood gases come back:

$$\begin{array}{lcll} P_{v_{CO_2}} & = & 48 & \text{mmHg} \\ P_{v_{O_2}} & = & 47 & \text{mmHg} \\ P_{a_{CO_2}} & = & 42 & \text{mmHg} \\ P_{a_{O_2}} & = & 60 & \text{mmHg} \end{array}$$

This time without caution notes.

What can you say about the cause of gas exchange impairment in this patient? (50%) Hint, you can ignore the oxygen carrying capacity of plasma in your calculations.

From alveolar gas eq:

$$\begin{aligned} P_{A_{O_2}} &\approx F_{I_{O_2}}\left(P_{atm} - P_{H_2O}\right) - \frac{P_{a_{CO_2}}}{RQ} \\ &= 1(713) - \frac{42}{0.8} = 660 \gg 60 \end{aligned}$$

Given that the paitent is breathing at $F_{I_{O_2}} = 1$, *this large* $660 - 60 = (A - a)_{D_{O_2}}$ *gradient can only be explained by shunt.*

Using the approximate shunt equation

$$\frac{Q_s}{Q_t} = \frac{1 - sat_{a,O_2}}{1 - sat_{v,O_2}} = \frac{1 - 0.9}{1 - 0.82} = \frac{0.1}{0.18} = 0.56$$

This patient has a 56% shunt fraction.

Problem 2 (Case 2)

The same patient eventually develops respiratory failure and is placed on a mechanical ventilator adjusted to parameters matching his tidal breathing:

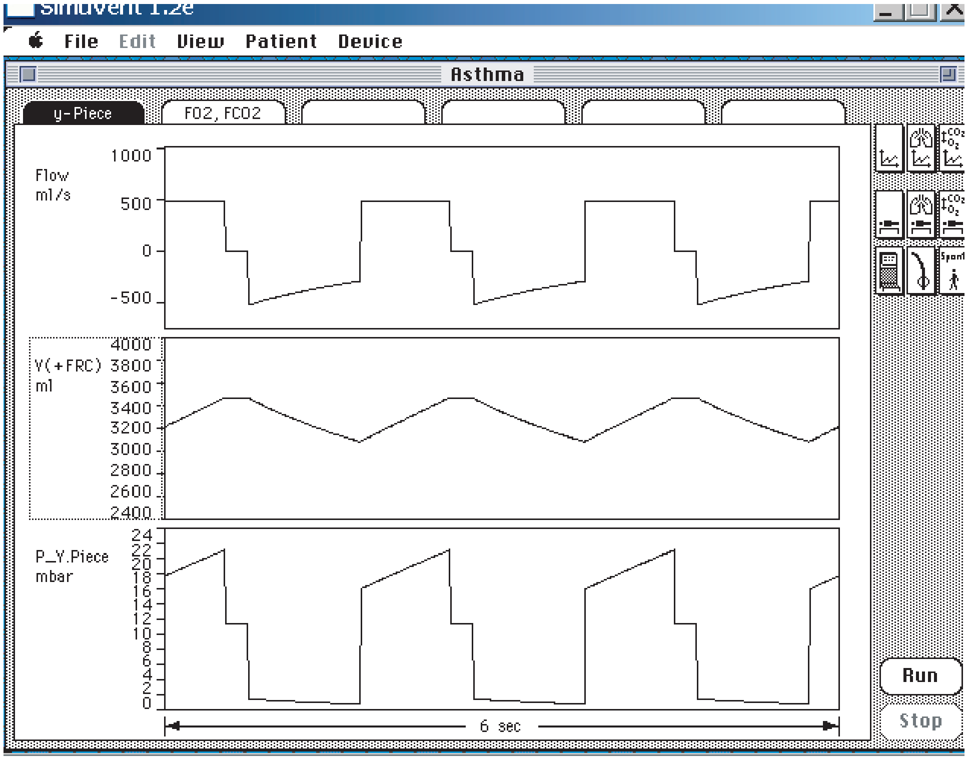
$$VT = 390 \text{ ml} \qquad f = 30 \text{ bpm} \qquad T_{ins} = 40\% \qquad T_{exp} = 50\% \qquad F_{i\mathrm{O}_2} = 0.50$$

And his blood gases are measured as:

$$\begin{array}{lcrl} P_{v\mathrm{CO}_2} & = & 42 & \text{mmHg} \\ P_{v\mathrm{O}_2} & = & 45 & \text{mmHg} \\ P_{a\mathrm{CO}_2} & = & 40 & \text{mmHg} \\ P_{a\mathrm{O}_2} & = & 275 & \text{mmHg} \\ & & & \\ \dot{V}_{\mathrm{O}_2} & = & 274 & \text{ml/min} \\ \dot{V}_{\mathrm{CO}_2} & = & 220 & \text{ml/min} \end{array}$$

The ventilator output shows the following screen

Figure 2:



A. Is this patient exhibiting dynamic hyper-inflation, and why or why not? (25%)

Yes, because expiratory flow at the end of exhalation is non-zero.

B. Can you estimate the patient's respiratory system mechanical parameters: Resistance and Compliance? (25%)

$$
\begin{aligned}
R &= \frac{P_{peak} - P_{plateau}}{\dot{V}} = \frac{21 - 11}{\frac{V_T}{t_{insp}}} = \frac{10}{\left(\frac{390}{2\times 0.4}\right)} \\
&= 2.05 \ mbar{\cdot}s/L \\
C &= \frac{V_T}{P_{peak - P_{ini}}} = \frac{390}{21 - 16} = 78 \ ml/mbar \\
RC &= 78 \times 0.0205 = 1.6 \ sec
\end{aligned}
$$

C. The attending MD suggests decreasing frequency while keeping the inspiration (insufflation in Germanic English) and exhalation time % unchanged. What frequency and tidal volume would you choose? Assume that the VD physiologic remains unchanged. (50%)

(Note: if you decide to use VD anatomic in your calculation, you will lose 25% of the question points.)

To avoid hyperinflation:

$$\begin{aligned} t_{exp} &\geq 4 \times RC = 4 \times 1.6 = 6.4\ sec \\ t_{exp}\% = 50\% &\Rightarrow f_2 = \frac{1}{\frac{6.4}{0.5}} \times 60 = 4.7\ bpm \end{aligned}$$

To keep a constant $\dot{V}_A$, we need to calculate a V_T such that

$$\begin{aligned} \left(V_{T_1} - V_D\right) f_1 &= \left(V_{T_2} - V_D\right) f_2 \\ V_{T_2} &= \left(V_{T_1} - V_D\right) \frac{f_1}{f_2} + V_D \end{aligned}$$

We know that

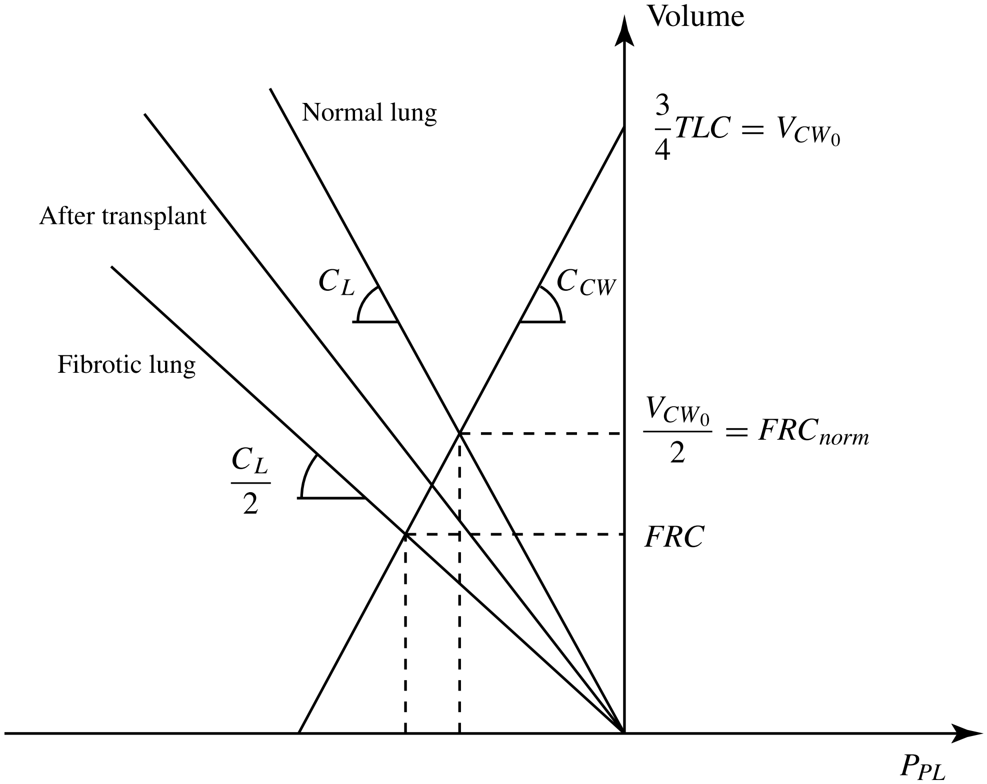
$$\frac{V_D}{V_T} = 1 - \frac{V_{CO_2}}{V_T\, f\, F_{A_{CO_2}}} \qquad where\ F_{A_{CO_2}} \approx \frac{P_{a_{CO_2}}}{(P_{atm} - 47)}$$

$$\begin{aligned} \frac{V_D}{V_T} &= 1 - \frac{220}{390 \times 30 \times \frac{40}{713}} = 0.665 \\ V_D &= 0.665 \times 390 = 259.35\ ml \\ V_{T_2} &= (390 - 259.35)\frac{30}{4.7} + 290.35 \\ &= 1124\ ml \\ V_{T_2} &= 1.124\ L \\ f_2 &= 4.7\ bpm \end{aligned}$$

Problem 3

Pulmonary fibrosis is a debilitating disease of the lung characterized by replacement of elastin by collagen and resulting in a decrease of lung compliance. In severe cases, lung transplant is the only option for survival. To maximize organ availability and reduce post-operative mortality, usually unilateral lung transplant is conducted.

A. First draw the normal chest wall and lung compliance curves. Then draw changes that result from pulmonary fibrosis (C_L reduced by 1/2). Assume that compliances are linear and that the chest wall compliance does not change. What happens with FRC in pulmonary fibrosis? (25%)



For CW:

$$V_{CW} = V_{CW_0} + P_{PL}C_{CW} \rightarrow P_{PL} = \frac{V_{CW} - V_{CW_0}}{C_{CW}}$$

For lungs:

$$V_L + \left(\underbrace{P_{a_0}}_{=0} - P_{PL} \right) C_L = -P_{PL}C_L \rightarrow P_{PL} = \frac{-V_L}{C_L}$$

FRC = Lung Volume, where $V_L = V_{CW}$ *and* $P_{PL_{CW}} = P_{PL_L}$.

$$\Rightarrow \frac{V_{CW} - V_{CW_0}}{C_{CW}} = \frac{-V_L}{C_L}$$

For $V_L = V_{CW} = FRC$

$$\begin{aligned} FRC - V_{CW_0} &= -FRC\frac{C_{CW}}{C_L} \\ FRC &= \frac{V_{CW_0}}{1 + \frac{C_{CW}}{C_L}} \end{aligned}$$

For normal lung

$$C_{CW} = C_L \rightarrow FRC = \frac{V_{CW_0}}{2}$$

For fibrotic lung

$$C_{L_F} = \frac{C_L}{2} \rightarrow FRC = \frac{V_{CW_0}}{1 + \frac{1}{1/2}} + \frac{V_{CW_0}}{3}$$

B. Second, draw the effects of replacing one of the lungs with a normal donor lung. What will be the new FRC after surgery? You can assume that both right and left lungs have equal compliance before surgery. (25%)

After transplant:

$$C_{L,post} = \underbrace{\frac{C_L}{4}}_{\substack{\uparrow \\ \textit{fibrotic} \\ \textit{lung}}} + \underbrace{\frac{C_L}{2}}_{\substack{\uparrow \\ \textit{transplanted} \\ \textit{“normal”} \\ \textit{lung}}} = \frac{3}{4}C_L$$

So

$$FRC = \frac{V_{CW_0}}{1 + \frac{4}{3}} = \frac{3}{7}V_{CW_0}$$

C. How does the amount of pressure required to inspire a similar tidal volume compare between before and after surgery? (25%)

Total lung compliance

Before transplant:

$$C_{L_F} = \frac{1}{2}C_L = \frac{V_T}{\Delta P} \Rightarrow \Delta P = \frac{V_T}{\left(\frac{C_L}{2}\right)}$$

After transplant:

$$\begin{aligned} C_{L,post} &= \frac{C_{L_F}}{2} + \frac{C_L}{2} = \frac{C_L}{4} + \frac{C_L}{2} = \frac{3}{4}C_L \\ \Delta P_{after} &= \frac{V_T}{\frac{3}{4}C_L} \rightarrow \Delta P_{after} = \frac{4}{3}\left(\frac{V_T}{C_L}\right) \\ \Delta P_{before} &= \frac{V_T}{\left(\frac{C_L}{2}\right)} = 2\left(\frac{V_T}{C_L}\right) \end{aligned}$$

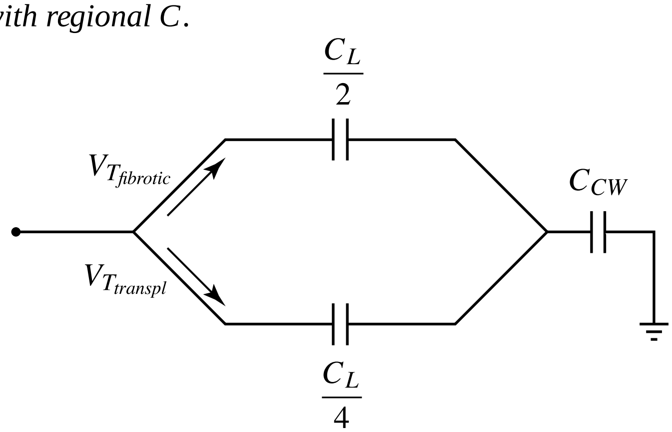
For the same V_T

$$\frac{\Delta P_{after}}{\Delta P_{before}} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

After transplant the person needs two-thirds the pleural pressure to generate the same total V_T.

D. In what proportions is the tidal volume distributed between both lungs? (25%)

V_T *distributes with regional* C.



$$\boxed{V_{T_{tranpl}} = 2V_{T_{fibrotic}}}$$