1.270J Logistics and Distribution Systems

Inventory and EOQ Models

Agenda

- Inventory
  - Reasons for holding inventory
  - Dimensions of inventory models
- EOQ-type models
  - Basic model
  - EPQ model
  - Planned backorders
  - Quantity discounts

Inventory

Inventory = cumulative supply – cumulative demand

Why hold inventories?

- The transaction motive
  - Economies of scale: production, transportation, discount, replenishment, ...
  - Competition purpose
- The precautionary motive
  - Demand uncertainty: unpredictable events
  - Supply uncertainty: lead time, random yield, ...
- The speculative motive
  - Fluctuating value: ordering cost, selling price
  - Demand increase: seasonality, promotion, ...

Dimensions of inventory models

- Products
  - single product vs. multiple products
  - perishable or durable
- Decision variables
  - when and how much to order
  - pricing
  - production and/or delivery schedule
  - capacity expansion
  - setup reduction
  - quality improvement
- Decision making structure
  - single decision maker vs. multiple decision makers
- Time
  - single period, finite horizon, infinite horizon
  - deterministic or stochastic

Dimensions of inventory models

- Objective function
  - costs (average or discounted): order/production, inventory holding and shortage
  - Profit
  - risk-neutral vs. risk averse
- Physical system
  - single location vs. multiple locations
  - single stage vs. multiple stages
- Information structure
  - continuous review vs. periodic review
  - inexact stock level
- Resource constraints
  - limited capacity

Dimensions of inventory models

- Supply
  - Controllable: when and how much to order
  - Supply contracts
  - Imperfect quality
  - Limited capacity
  - Lead time
- Demand
  - Exogenous: deterministic (constant or time dependent), stochastic
  - Endogenous: pricing model

Ordering costs in inventory models

- Ordering costs
  - Linear: proportional to order quantity
  - Concave: economies of scale, incremental discount
  - General: all-units discount

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Inventory costs in inventory models

- Inventory carrying costs
  - Insurance cost: 2%
  - Maintenance cost: 6%
  - Opportunity cost of alternative investment:7-10%
- Shortage costs: loss of good will or reputation (hard to quantify)
  - Lost sale case
  - Backorder case

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EOQ Model: Assumptions

- infinite horizon
- constant and deterministic demand: D items/unit time
- no shortages
- fixed order quantity Q
- zero lead time
- order cost: $K + cQ$
- inventory holding cost: h per item per unit time

Objective of the EOQ model

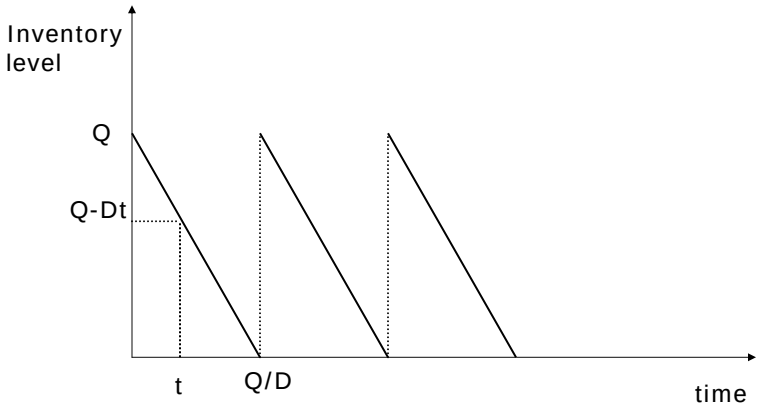
- Objective: minimize the average cost per unit of time over the infinite horizon subject to no shortages

$$\min \lim_{T \to \infty} \frac{1}{T} \int_0^T \{hI(t) + O(t)\} dt$$

$I(t)$: inventory level at time t

$O(t)$: order cost at time t

EOQ Model: Graphical Representation



EOQ Model: Costs per unit time

the total cost for one cycle
= the ordering cost+inventory holding cost
$= (K + cQ) + h\int_0^{Q/D} I(t)dt$
$= (K + cQ) + h\int_0^{Q/D} (Q - Dt)dt$
$= (K + cQ) + \frac{hQ^2}{2D}$

the average cost per unit of time
= the total cost for one cycle/cylce time
$= \frac{KD}{Q} + \frac{h}{2}Q + cD$

EOQ Model: EOQ

Find the optimal ordering quantity:
Economic Order Quantity

First order optimality condition
$0 = \frac{d(\text{Cost per unit time})}{dQ} = -\frac{KD}{Q^2} + \frac{h}{2}$

EOQ=$Q^* = \sqrt{\frac{2DK}{h}}$
Optimal Cycle Time=$T^* = \sqrt{\frac{2K}{hD}}$

EOQ Model: Two Questions

- Lead Time
- Why EOQ independent of the variable ordering cost?

EOQ Model: One Example

A) The demand for electrical components is fixed at a rate of 2400 units/month. Each time the store makes an order it costs 320$. The item costs 3$. The annual invenotry holding cost rate is 20%.

$Q^*=5543$ units, $T^*=2.3$ months

EOQ Model: One Example

B) Suppose we now order electrical components only in hundreds of units.

$Q^*=5500$ units
$C(5500)=277.13+cD$ $
$C(5600)=278+cD$ $

Sensitivity Analysis

$$
\begin{aligned}
C(Q^*) &= \sqrt{2KDh} \\
C(\gamma Q^*) &= \frac{1}{\gamma}\sqrt{\frac{1}{2}KDh} + \gamma\sqrt{\frac{1}{2}KDh} \\
&= \sqrt{2KDh}(\gamma + \tfrac{1}{\gamma})/2 \\
\frac{C(\gamma Q^*)}{C(Q^*)} &= (\gamma + \tfrac{1}{\gamma})/2
\end{aligned}
$$

γ	0.5	0.8	0.9	1	1.2	1.5	2
$\frac{C(\gamma Q^*)}{C(Q^*)}$	1.25	1.025	1.006	1	1.017	1.083	1.25

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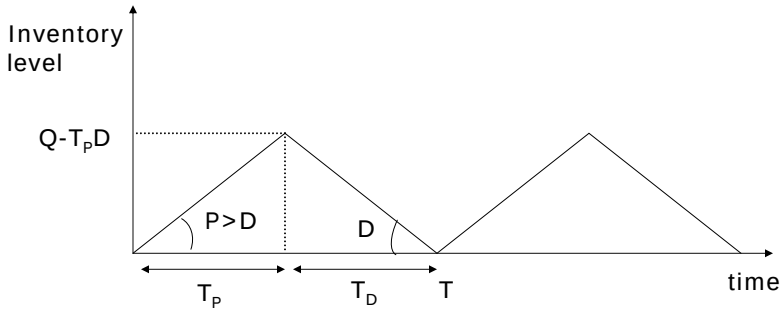
EPQ Model

- Production Planning Model
  - No shortages
  - Production rate: P>D



EPQ: Analysis

Let the total demand in one cycle is $Q = T_P P$
The cost for one cycle
$= K + h\int_0^{T_P}(P-D)tdt + h\int_{T_P}^{T}(Q-Dt)dt$
$= K + \frac{h}{2}(P-D)\frac{Q^2}{P^2} + h[\frac{Q^2}{2D} - \frac{Q^2}{P} + \frac{D}{2}\frac{Q^2}{P^2}]$

Average cost per unit time $C(Q)$
$= \frac{DK}{Q} + \frac{hQ}{2}\frac{P-D}{P}$

Economic Production Quantity:
$\frac{dC(Q)}{dQ} = 0 \rightarrow Q^* = \sqrt{\frac{2DK}{h}}\sqrt{\frac{P}{P-D}}$
$C(Q^*) = \sqrt{2DKh}\sqrt{\frac{P-D}{P}}$

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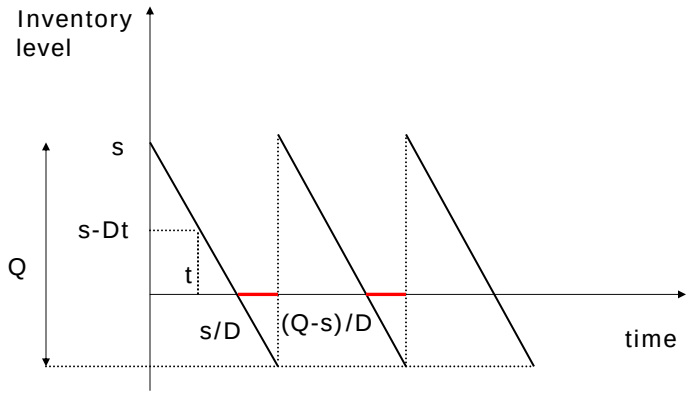
EOQ Model: Planned Backorder

- Let π be the shortage cost per item per unit of time



EOQ Model: Backorder Analysis

Total Costs Per Cycle

$$TC(s,Q) = K + \frac{hs^2}{2D} + \frac{\pi}{2D}(Q-s)^2$$

Average Cost per unit time

$$C(s,Q) = \frac{DK}{Q} + \frac{hs^2}{2Q} + \frac{\pi}{2Q}(Q-s)^2$$

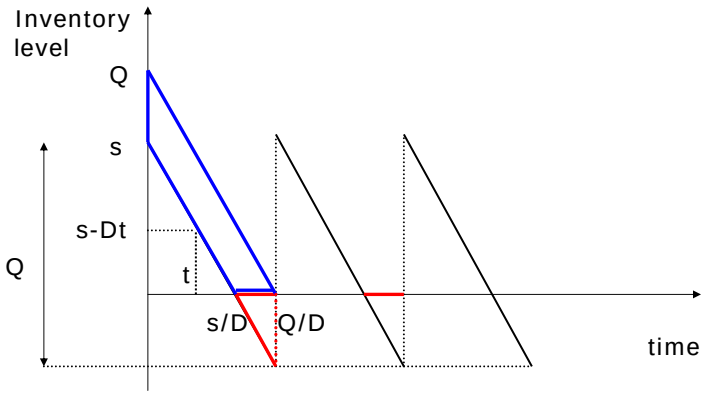
Economic Order Quantity and Order-up-to Level

$$Q^* = \sqrt{\frac{2KD}{h}}\sqrt{\frac{\pi+h}{\pi}}$$

$$s^* = \sqrt{\frac{2KD}{h}}\sqrt{\frac{\pi}{\pi+h}}$$

$$C(s^*,Q^*) = \sqrt{2KDh}\sqrt{\frac{\pi}{\pi+h}}$$

Why shortages?



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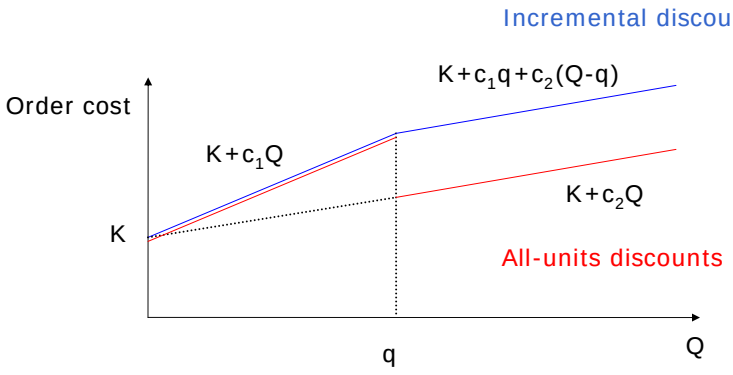
Quantity Discounts



$c_1 > c_2$

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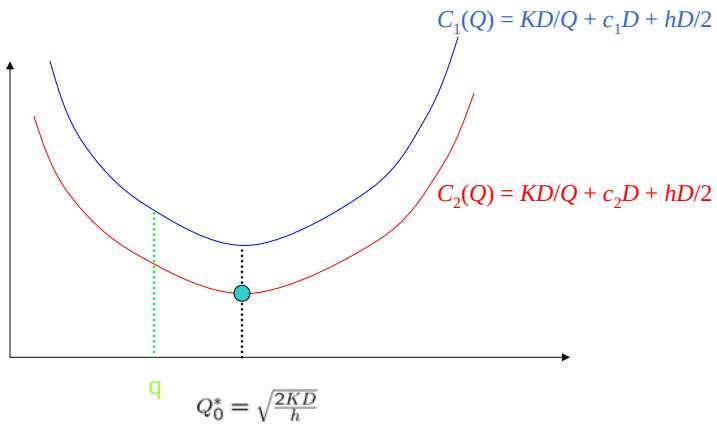
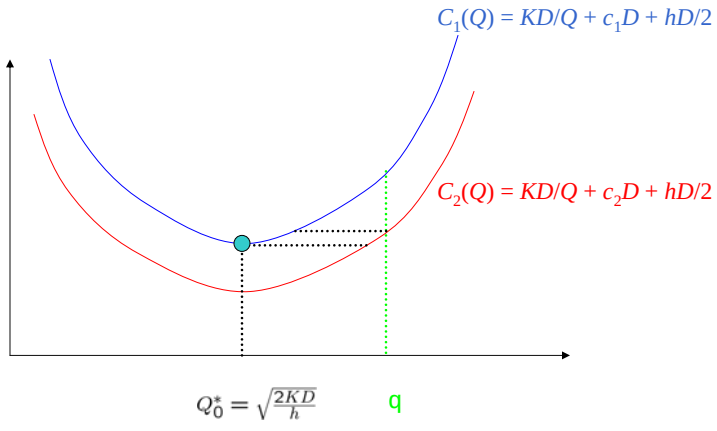
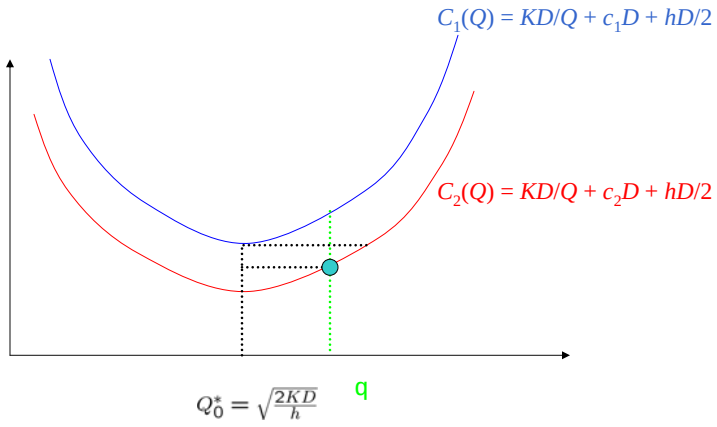
All-units discounts: Case 1



All-units discounts: Case 2



All-units discounts: Case 3



All-units discount: Summary

Case 1: $q \leq Q_0^*,\ Q^* = Q_0^*$

Case 2: $q \geq Q_0^*,\ C_1(Q_0^*) \leq C_2(q),\ Q^* = Q_0^*$

Case 3: $q \geq Q_0^*,\ C_1(Q_0^*) \geq C_2(q),\ Q^* = q$

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