Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.977 Ultrafast Optics

Spring 2005

Problem Set 4

Issued: Mar. 3, 2005. **Due:** March 10, 2005.

Problem 4.1: Numerical Solution of the NSE

The NSE can easily be numerically solved using the Split-Step Fourier transform. Starting from the NSE

$$\frac{\partial A(z,t)}{\partial z} = jD_2 \frac{\partial^2 A(z,t)}{\partial t^2} - j\delta |A(z,t)|^2 A(z,t), \tag{1}$$

(a) Show by using the following transform

$$\begin{aligned} \xi &= z/l_D, \qquad \tau = t/\tau_0 \\ l_D &= \frac{\tau_0^2}{|D_2|}, \qquad u = \sqrt{\frac{\delta}{|D_2|}}\tau_0 A, \end{aligned}$$

that the NSE can be broad into the normalized form

$$\frac{\partial u(\xi,\tau)}{\partial \xi} = -j \frac{\partial^2 u(\xi,\tau)}{\partial \tau^2} - j|u(\xi,\tau)|^2 u(\xi,\tau), \tag{2}$$

(b) The NSE can be understood in the following way

$$\frac{\partial u(\xi,\tau)}{\partial \xi} = \left(\hat{D} + \hat{N}\right) u(\xi,\tau), \tag{3}$$

as the simultaneous action of a dispersion operator $\hat{D} = -j\frac{\partial^2}{\partial \tau^2}$, and a nonlinear operator $\hat{N} = -j|u(\xi,\tau)|^2$. If the linear and nonlinear changes in the pulse are small within a short distance of propagation $\Delta\xi$, then, the solution of the NSE, which can be symbolically written as

$$u(\Delta\xi,\tau) = e^{\left(\hat{D}+\hat{N}\right)\Delta\xi} u(0,\tau) \tag{4}$$

and approximated by

$$u(\Delta\xi,\tau) = e^{\frac{1}{2}\hat{D}\Delta\xi} e^{\hat{N}\Delta\xi} e^{\frac{1}{2}\hat{D}\Delta\xi} u(0,\tau).$$

One can show that iterative application of this propagation step only leads to an error of order $\Delta\xi^3$. Since the linear operator can be easily applied in the Fourier domain and the nonlinear operator (only SPM) in the time domain, one can simulate the NSE over one propagation step $\Delta\xi$ by the following algorithm

$$u(\xi \pm \Delta\xi,\tau) = F^{-1}\left[e^{\frac{1}{2}j\omega^2\Delta\xi} F\left[e^{-j|u(\xi,\tau)|^2\Delta\xi} F^{-1}\left[e^{\frac{1}{2}j\omega^2\Delta\xi} F\left[u(\xi,\tau)\right]\right]\right]\right].$$

Write a program in some programming language you are familiar with or MATLAB and simulate the NSE for the following initial pulses

$$u(0,\tau) = N\ \text{sech}(\frac{\tau}{\sqrt{2}}). \tag{5}$$

for N=1, 2 and evtually 3. Make use of the Fast Fourier Transform (FFT) and use at least 1024 points. Plot the pulse shape (in the time domain) and corresponding amplitude spectra (in the frequency domain) as a function of propagation distance, similar to those shown in Chapter 3.

Problem 4.2: Rate Equations for the Four-Level Laser

As an example for a four-level laser material we consider the transition of Nd:YAG at $\lambda_0 = 1.064\,\mu m$. With a diode laser at $0.8\,\mu m$ the $^4F_{5/2}$ level can be pumped from the ground state $^4I_{9/2}$. We assume that fast relaxation rates lead to efficient excitation transfer from the pump level to the upper laser level $^4F_{3/2}$ and from the lower laser level $^4I_{11/2}$ to the ground state $^4I_{9/2}$, i.e. we assume $\gamma_{32}, \gamma_{10} \gg \gamma_{21}$. The cross section for stimulated emission between level 2 and 1 is given by $\sigma_{12} = \sigma = 6.5 \times 10^{-19} cm^2$. The pump rate R_P is assumed to be constant. The transition rates between the levels are denoted by $\gamma_{ij} = \frac{1}{T_{ij}}$.

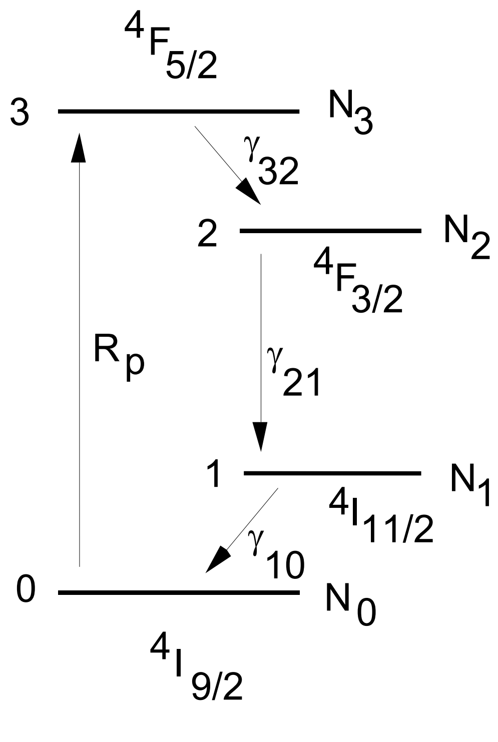

Figure 1: The four levels in Nd:YAG related to lasing at 1064 nm

(a) Write down the four rate equations for the four-level laser system shown in Figure 1 using the level populations N_2 and the photon density n_L, cross section σ and the group velocity v_g of the laser light.

(b) Write down the rate equations for the photon density n_L and intensity $I \propto n_L$ in the resonator mode. For simplicity assume that the effective cross section of the laser mode A_{eff} is same everywhere in the resonator. The resonator length is L.

(c) In general γ_{32}, γ_{10} are fast relaxation rates much faster than any other processes in the system. Therefore, the populations N_3 and N_1 always relax quasi instantaneously into the steady state determined by the values for intensity I and population N_2. Eliminate the fast populations N_3 and N_1 and derive a rate equation for $N_2 - N_1$. Assume that the population in the ground state never gets depleted.