6.852: Distributed Algorithms
Fall, 2009

Class 4

Today's plan

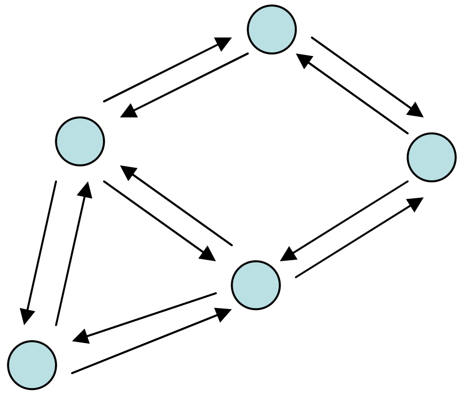
- Fault-tolerant consensus in synchronous systems
- Link failures:
  - The Two Generals problem
- Process failures:
  - Stopping and Byzantine failure models
  - Algorithms for agreement with stopping and Byzantine failures
  - Exponential information gathering
- Reading: Section 5.1, 6.1-6.3
- Next:
  - Lower bounds for Byzantine agreement:
    - Number of processors
    - Number of rounds
  - Reading:
    - Sections 6.4-6.7
    - [Aguilera, Toueg]
    - (Optional) [Keidar-Rajsbaum]

Distributed consensus

- Abstract problem of reaching agreement among processes in a distributed system, all of which start with their own “opinions”.
- Complications: Failures (process, link); timing uncertainties.
- Motivation:
  - Database transactions: Commit or abort
  - Aircraft control:
    - Agree on value of altimeter reading (SIFT)
    - Agree on which plane should go up/down, in resolving encounters (TCAS)
  - Resource allocation: Agree on who gets priority for obtaining a resource, doing the next database update, etc.
  - Replicated state machines: To emulate a virtual machine consistently, agree on next step.
- Fundamental problem
- We’ll revisit it several times:
  - In synchronous, asynchronous, and partially synchronous settings.
  - With link failures, processor failures.
  - Algorithms, impossibility results.

Consensus with link failures

- Informal scenario:
  - Several generals plan a coordinated attack.
  - All should agree to attack:
    - Absolutely must agree.
    - Should attack if possible.
  - Each has an initial opinion about his army's readiness.
  - Nearby generals can communicate using foot messengers:
    - Unreliable, can get lost or captured
    - Connected, undirected communication graph, known to all generals, known bound on time for successful messenger to deliver message.
- Motivation: Transaction commit
- Can show no algorithm exists!



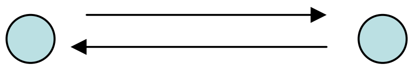
Formal problem statement

- G = (V,E), undirected graph (bidirected edges)
- Synchronous model, n processes
- Each process has input 1 (attack) or 0 (don’t attack).
- Any subset of the messages can be lost.
- All should eventually set decision output variables to 0 or 1.
  - In practice, would need this by some deadline.
- Correctness conditions:
  - Agreement:
    - No two processes decide differently.
  - Validity:
    - If all start with 0, then 0 is the only allowed decision.
    - If all start with 1 and all messages are successfully delivered, then 1 is the only allowed decision.

Alternatively:

- Stronger validity condition:
  - If anyone starts with 0 then 0 is the only allowed decision.
  - If all start with 1 and all messages are successfully delivered, then 1 is the only allowed decision.
  - Typical for transaction commit (1 = commit, 0 = abort).
- Guidelines:
  - For designing algorithms, try to use stronger correctness conditions (better algorithm).
  - For impossibility results, use weaker conditions (better impossibility result).

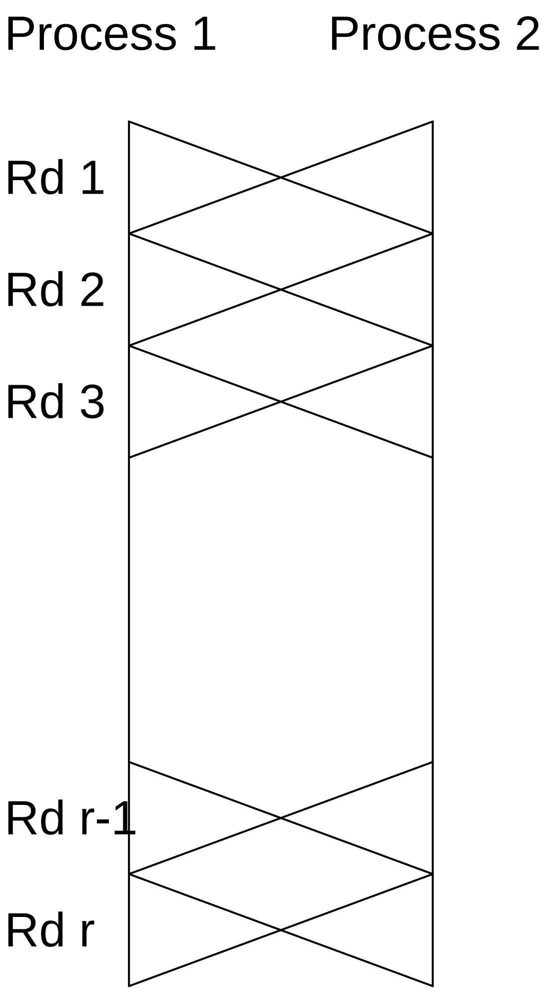
Impossibility for 2 Generals [Gray]



- Other cases similar, LTTR.
- Proof: By contradiction.
  - Suppose we have a solution---a process (states, transitions) for each index 1, 2.
  - Assume WLOG that both processes send messages at every round.
    - Could add dummy messages.
  - Proof based on limitations of local knowledge.
  - Start with α, the execution where both start with 1 and all messages are received.
    - By the termination condition, both eventually decide.
    - Say, by the end of r rounds.
    - By the validity condition, both decide on 1.

2-Generals Impossibility

- α_1: Same as α, but lose all messages after round r.
  - Doesn't matter, since they've already decided by round r.
  - So, both decide 1 in α_1.
- α_2: Same as α_1, but lose the last message from process 1 to process 2.
  - Claim α_1 is indistinguishable from α_2 by process 1, $\alpha_1 \sim^1 \alpha_2$.
  - Formally, 1 sees the same sequence of states, incoming and outgoing messages.
  - So process 1 also decides 1 in α_2.
  - By termination, process 2 decides in α_2.
  - By agreement, process 2 decides 1 in α_2.

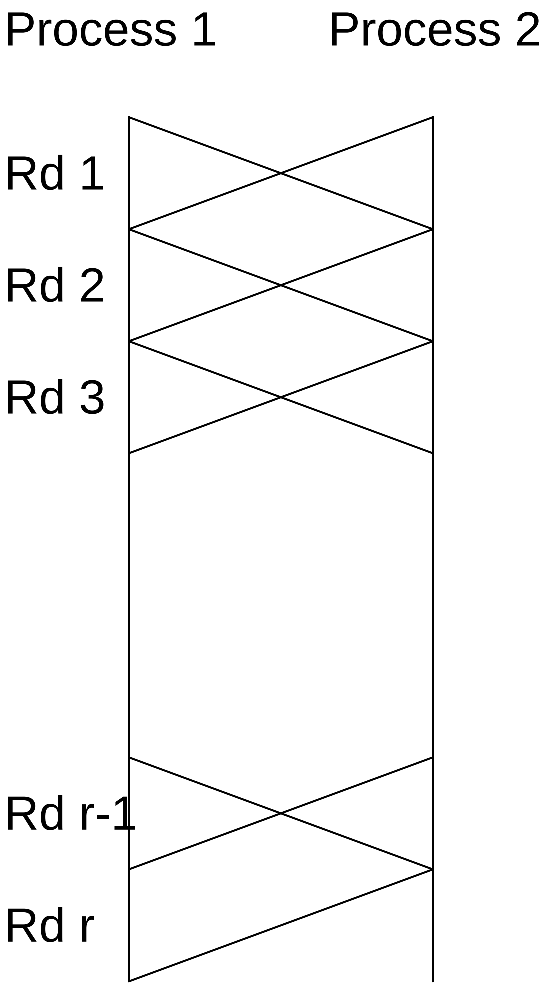


A fine point:

- In α_2 , process 2 must decide 1 at some point, not necessarily by round r.

Continuing…

- α_3: Same as α_2, but lose the last message from process 2 to process 1.
  - Then $\alpha_2 \sim^2 \alpha_3$.
  - So process 2 decides 1 in α_3.
  - By termination, process 1 decides in α_3.
  - By agreement, process 1 decides 1 in α_3.
- α_4 : Same as α_3, but lose the last message from process 1 to process 2.
  - Then $\alpha_3 \sim^1 \alpha_4$.
  - So process 1 decides 1 in α_4.
  - So process 2 decides 1 in α_4.
- Keep removing edges, get to:



The contradiction

- α_{2r+1} : Both start with 1, no messages received.
  - Still both must eventually decide 1.
- α_{2r+2} : process 1 starts with 1, process 2 starts with 0, no messages received.
  - Then $\alpha_{2r+1} \sim^1 \alpha_{2r+2}$.
  - So process 1 decides 1 in α_{2r+2}.
  - So process 2 decides 1 in α_{2r+2}.
- α_{2r+3} : Both start with 0, no messages received.
  - Then $\alpha_{2r+2} \sim^2 \alpha_{2r+3}$.
  - So process 2 decides 1 in α_{2r+3}.
  - So process 1 decides 1 in α_{2r+3}.

- But α_{2r+3} contradicts weak validity!

Consensus with process failures

- Stopping failures (crashes) and Byzantine failures (arbitrary processor malfunction, possibly malicious)
- Agreement problem:
  - n-node connected, undirected graph, known to all processes.
  - Input v from a set V, in some state variable.
  - Output v from V, by setting decision := v.
  - Bounded number $\leq$ f of processors may fail.
- Bounded number of failures:
  - A typical way of describing limited amounts of failure.
  - Alternatives: Bounded rate of failure; probabilistic bounds on failure.

Stopping agreement

- Assume process may stop at any point:
  - Between rounds.
  - While sending messages at a round; any subset of intended messages may be delivered.
  - After sending, before changing state.
- Correctness conditions:
  - Agreement: No two processes (failing or not) decide on different values.
    - "Uniform agreement"
  - Validity: If all processes start with the same v, then v is the only allowable decision.
  - Termination: All nonfaulty processes eventually decide.
- Alternatively:
  - Stronger validity condition: Every decision value must be some process' initial value.
  - Use this later, for k-agreement.

Byzantine agreement

- "Byzantine Generals Problem" [Lamport, Pease, Shostak]
  - Originally "Albanian Generals"
- Faulty processes may exhibit "arbitrary behavior":
  - Can start in arbitrary states, send arbitrary messages, perform arbitrary transitions.
  - But can't affect anyone else's state or outgoing messages.
  - Often called "malicious" (but they aren't necessarily).
- Correctness conditions:
  - Agreement: No two nonfaulty processes decide on different values.
  - Validity: If all nonfaulty processes start with the same v, then v is the only allowable decision for nonfaulty processes.
  - Termination: All nonfaulty processes eventually decide.

Technicality about stopping vs. Byzantine agreement

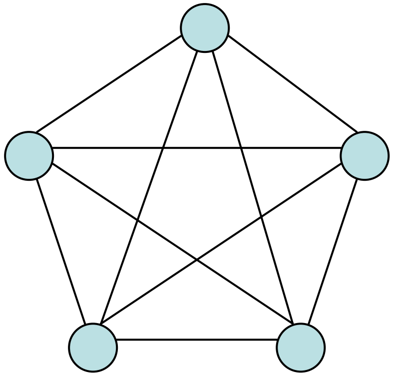
- A Byzantine agreement algorithm doesn’t necessarily solve stopping agreement:
- For stopping, all processes that decide, even ones that later fail, must agree (uniformity condition).
- Too strong for Byzantine setting.
- Implication holds in some special cases, e.g., when all decisions must happen at the end.

Complexity measures

- Time: Number of rounds until all nonfaulty processes decide.
- Communication: Number of messages, or number of bits.
  - For Byzantine case, just count those sent by nonfaulty processes.

Simple algorithm for stopping agreement

- Assume complete n-node graph.
- Idea:
  - Processes keep sending all V values they've ever seen.
  - Use simple decision rule at the end.
- In more detail:
  - Process i maintains $W \subseteq V$, initially containing just i's initial value.
  - Repeatedly: Broadcast W, add received elements to W.
  - After k rounds:
    - If $|W| = 1$ then decide on the unique value.
    - Else decide on a default value $v_0 \in V$.



- Q: How large should k be?

How many rounds?

- Depends on number f of failures to be tolerated.
- $f = 0$:
  - $k = 1$ is enough.
  - All get same W.
- $f = 1$:
  - $k = 1$ doesn't work:
    - Say process 1 has initial value u, others have initial value v.
    - Process 1 fails during round 1, sends to some and not others.
    - So some have $W = \{v\}$, others $\{u,v\}$, may decide differently.
  - $k = 2$ does work:
    - If someone fails in round 1, then no one fails in round 2.
- General f:
    - $k = f + 1$

Correctness proof (for $k = f+1$)

- Claim 1: Suppose $1 \leq r \leq f+1$ and no process fails during round r. Let i and j be two processes that haven't failed by the end of round r. Then $W_i = W_j$ right after round r.
- Proof: Each gets exactly the union of all the W's of the processes that have not failed by the beginning of round r.
- "Clean round"---allows everyone to resolve their differences.

- Claim 2: Suppose all the W sets are identical just after round r, for all processes that are still non-failed. Then the same is true for any $r' > r$.
- Proof: Obvious.

Check correctness conditions

- Agreement:
  - $\exists$ round r, $1 \leq r \leq f+1$, at which no process fails (since $\leq f$ failures)---a clean round.
  - Claim 1 says all that haven't yet failed have same W after round r.
  - Claim 2 implies that all have same W after round f + 1.
  - So nonfaulty processes pick the same value.
- Validity:
  - If everyone starts with v, then v is the only value that anyone ever gets, so |W| = 1 and v is chosen.
- Termination:
  - Obvious from decision rule.

Complexity bounds

- Time: f+1 rounds
- Communication:
  - Messages: $\leq (f + 1)\, n^2$
  - Message bits: Multiply by n b

Number of values sent in a message

A fixed bound on number of bits to represent a value in V.

- Can improve communication:
  - Messages: $\leq 2\, n^2$
  - Message bits: Multiply by b

Improved algorithm (Opt)

- Each process broadcasts its own value in round 1.
- May broadcast at one other round, just after it first learns about some value different from its own.
- In that case, it chooses just one such value to rebroadcast.
- After f + 1 rounds, use same rule as before:
  - If |W| = 1 then decide on the unique value.
  - Else decide on default value v_0.

Correctness

- Relate behavior of Opt to that of the original algorithm.
- Specifically, relate executions of both algorithms with the same inputs and same failure pattern.
- Let OW denote the W set in the optimized algorithm.
- Relation between states of the two algorithms:
  - For every i:
    - $OW_i \subseteq W_i$.
    - If $|W_i| = 1$ then $OW_i = W_i$.
    - If $|W_i| > 1$ then $|OW_i| > 1$.

Not necessarily the same set, but both > 1.

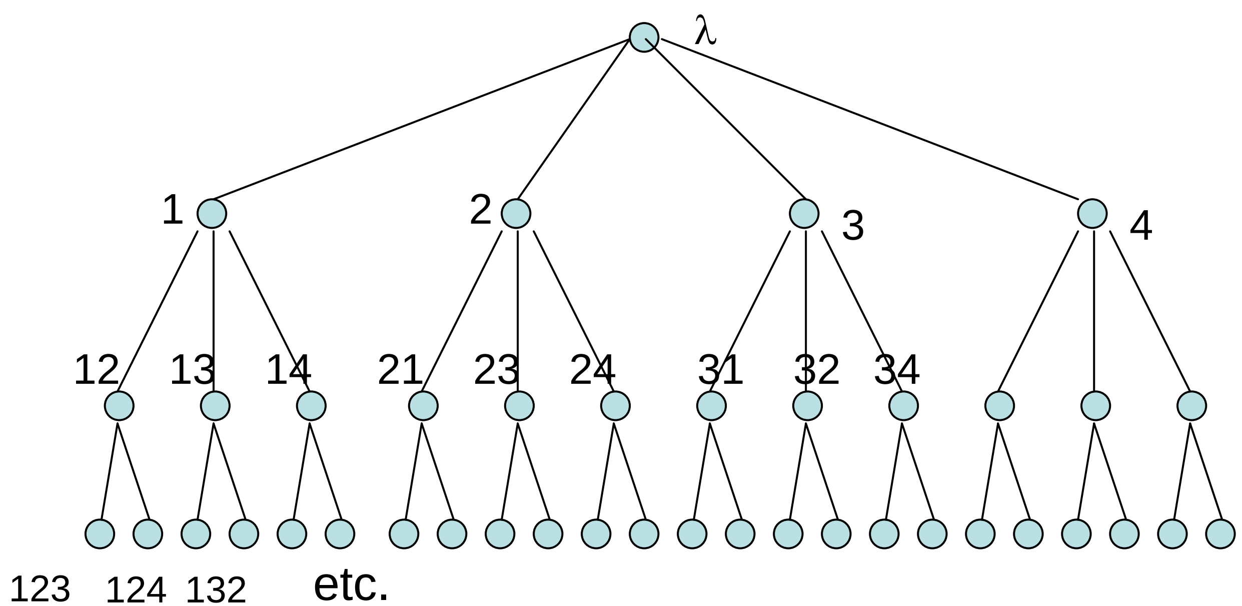
- Relation after f+1 rounds implies same decisions.

Proof of correspondence

- Induction on number of rounds (p. 107)
- Key ideas:
  - $OW_i \subseteq W_i$
    - Obvious, since Opt just suppresses sending of some messages from Unopt.
  - If $|W_i| = 1$ then $OW_i = W_i$.
    - Nothing suppressed in this case.
    - Actually, follows from the first property and the fact that OW_i is always nonempty.
  - If $|W_i| > 1$ then $|OW_i| > 1$.
    - Inductive step, for some round r:
    - If in Unopt, i receives messages only from processes with $|W| = 1$, then in Opt, it receives the same sets. So after round r, $OW_i = W_i$.
    - Otherwise, in Unopt, i receives a message from some process j with $|W_j| > 1$, and so (by induction), $|OW_j| > 1$. Then after round r, $|W_i| > 1$ and $|OW_i| > 1$.

Exponential Information Gathering (EIG)

- A strategy for consensus algorithms, which works for Byzantine agreement as well as stopping agreement.
- Based on EIG tree data structure.
- EIG tree $T_{n,f}$, for n processes, f failures:
  - f+2 levels
  - Paths from root to leaf correspond to strings of f+1 distinct process names.
- Example: $T_{4,2}$

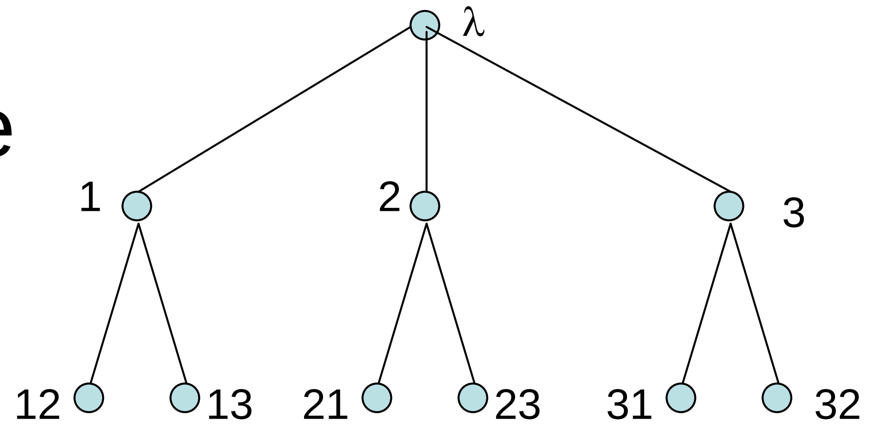


EIG Stopping agreement algorithm

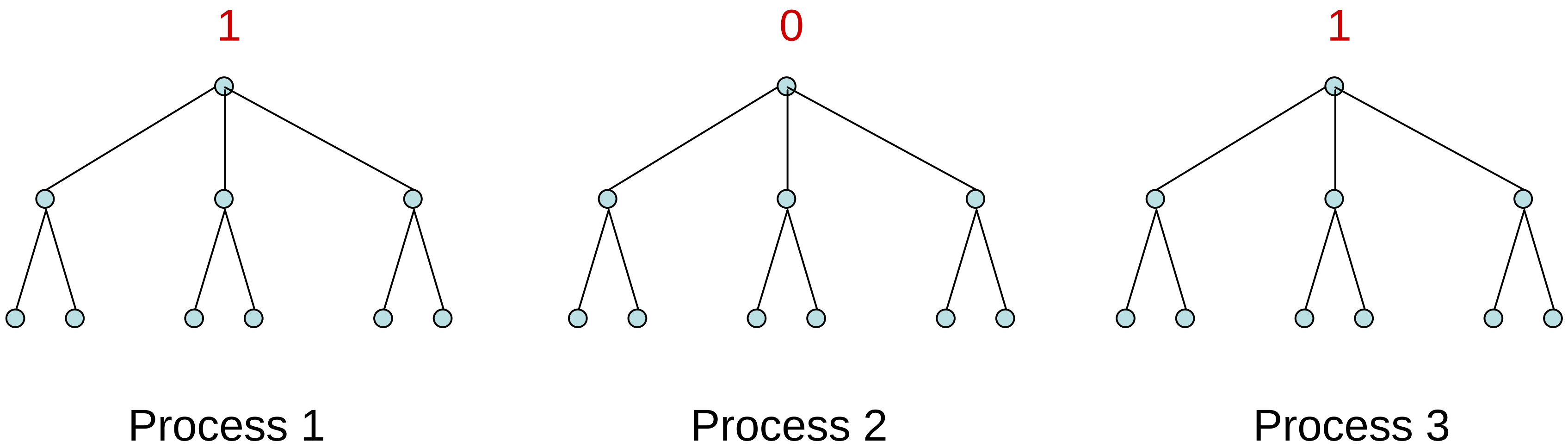
- Each process i uses the same EIG tree, $T_{n,f}$.
- Decorates nodes of the tree with values in V, level by level.
- Initially: Decorate root with i's input value.
- Round $r \geq 1$:
  - Send all level r-1 decorations for nodes whose labels don't include i, to everyone.
    - Including yourself---simulate locally.
  - Use received messages to decorate level r nodes---to determine label, append sender's id at the end.
  - If no message received, use $\perp$.
- The decoration for node $(i_1,i_2,i_3,\ldots,i_k)$ in i's tree is the value v such that (i_k told i) that (i_{k-1} told i_k) that …that (i_1 told i_2) that i_1's initial value was v.
- Decision rule for stopping case:
  - Trivial
  - Let W = set of all values decorating the local EIG tree.
  - If |W| = 1 decide that value, else default v_0.

Example

- 3 processes, 1 failure
- Use $T_{3,1}$:

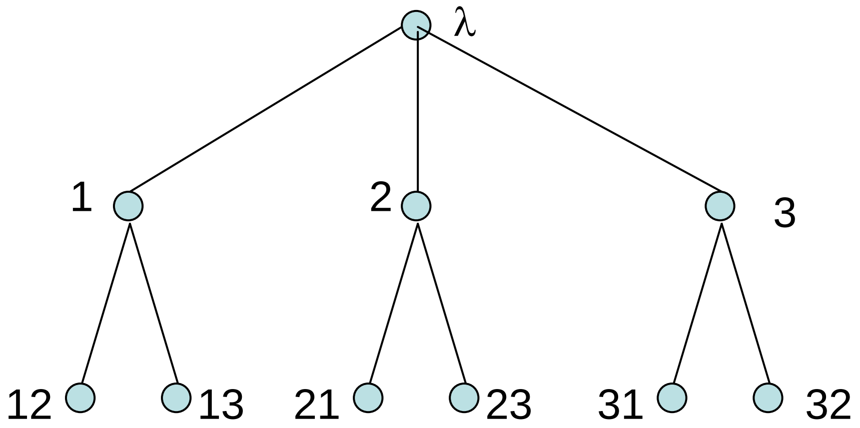


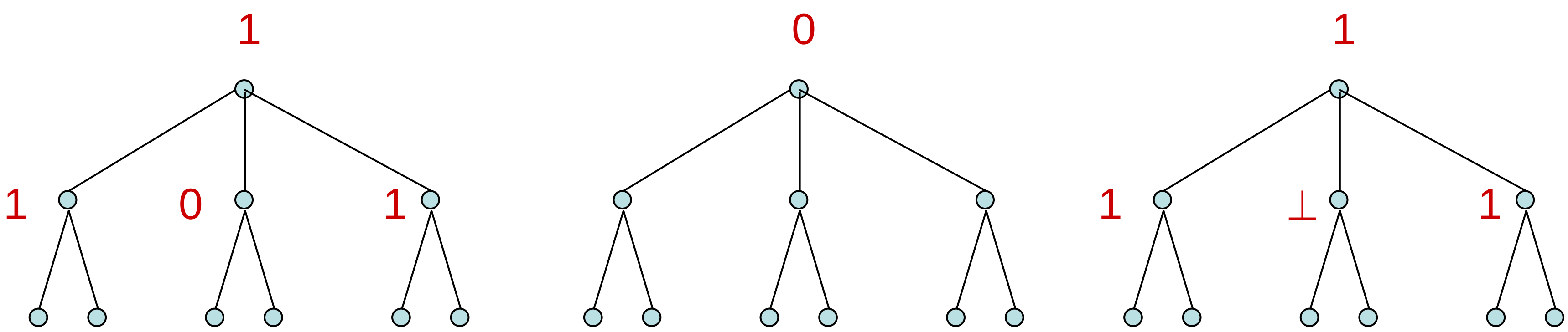
Initial values:



Example

- Process 2 is faulty, fails after sending to process 1 at round 1.
- After round 1:





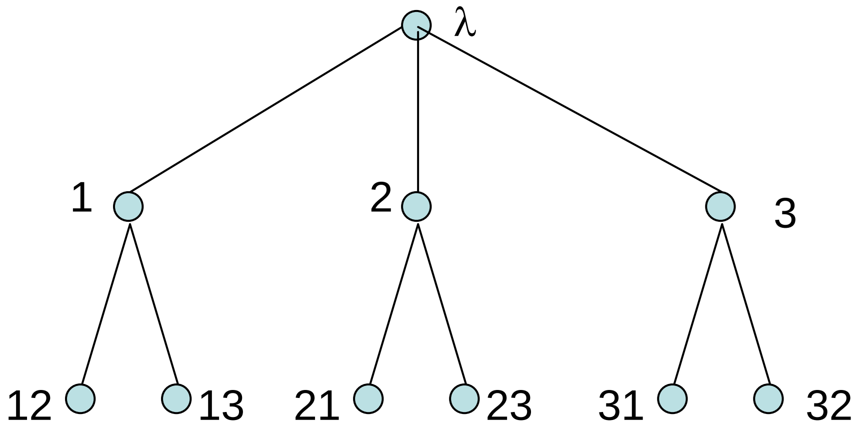
Process 1

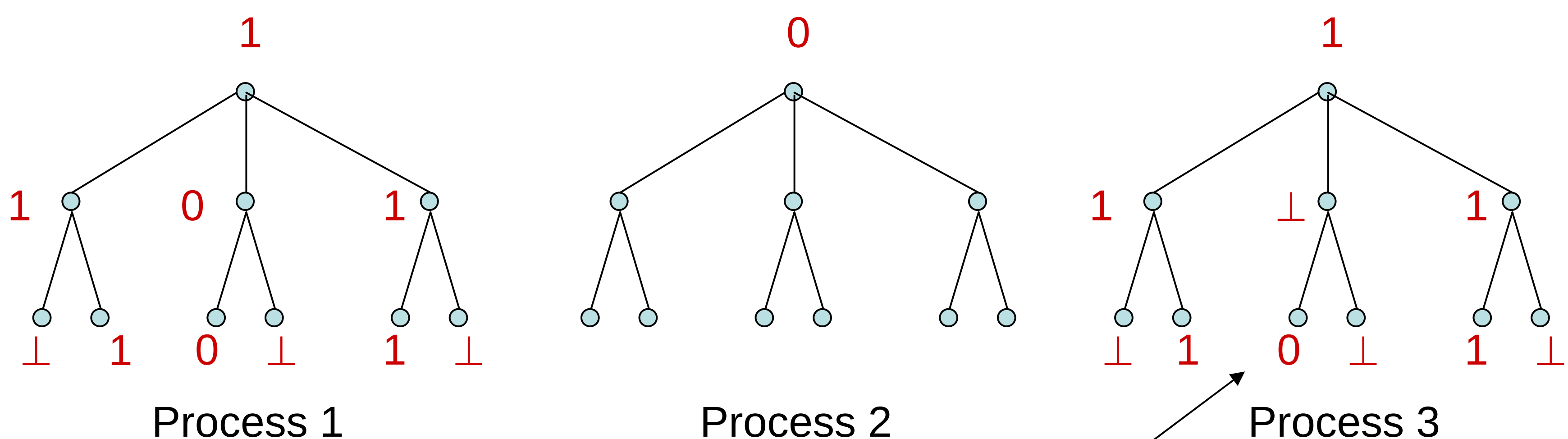
Process 2

Process 3

Example

- After round 2:





p3 discovers that p2’s value is 0 after round 2, by hearing it from p1.

Correctness and complexity

- Correctness similar to previous algorithms.
- Time: $f+1$ rounds, as before.
- Messages: $\leq (f + 1)\, n^2$
- Bits: Exponential in number of failures, $O(n^{f+1}\, b)$
- Can improve as before by only relaying the first two messages with distinct values.
- Extension:
  - The simple EIG stopping algorithm, and its optimized variant, can be used to tolerate worse types of failures.
  - Not full Byzantine model---that will require more work…
  - Rather, a restricted version of the Byzantine model, in which processes can authenticate messages.
  - Removes ability of process to relay false information about what other processes said.

Byzantine agreement algorithm

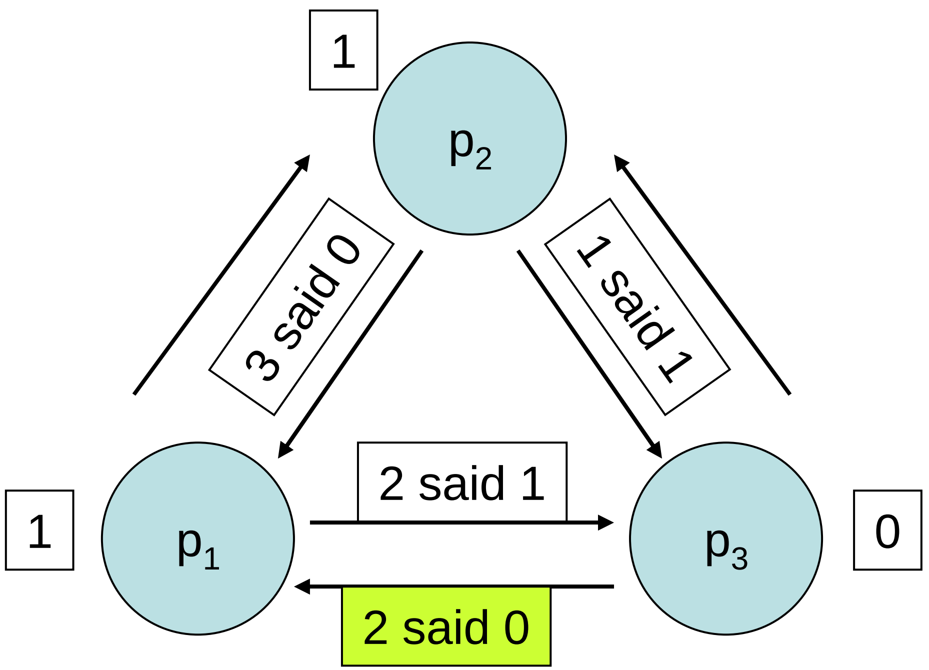
- Recall correctness conditions:
  - Agreement: No two nonfaulty processes decide on different values.
  - Validity: If all nonfaulty processes start with the same v, then v is the only allowable decision for nonfaulty processes.
  - Termination: All nonfaulty processes eventually decide.
- Present EIG algorithm for Byzantine agreement, using:
  - Exponential communication (in f)
  - f+1 rounds
  - n > 3f
- Expensive!
  - Time bound: Inherent. (Lower bound)
  - Number-of-processors bound: Inherent. (Lower bound)
  - Communication: Can be improved to polynomial.

Bad example: n = 3, f = 1

- Consider three executions of an EIG algorithm, with any decision rule.
- α_1: p1 and p2 nonfaulty, initial value 1, p3 faulty, initial value 0
  - Round 1: All truthful
  - Round 2: p3 lies, telling p1 that “p2 said 0”; all other communications are truthful.
  - Validity requires that p1 and p2 decide 1.
- α_2: p2 and p3 nonfaulty, initial value 0, p1 faulty, initial value 1
  - Round 1: All truthful
  - Round 2: p1 lies, telling p3 that “p2 said 1”; all other communications are truthful.
  - Validity requires that p2 and p3 decide 0.
- α_3: p1 nonfaulty, initial value 1, p3 nonfaulty, initial value 0, p2 faulty, initial value doesn’t matter.
  - Round 1: p2 tells p1 its initial value is 1, tells p3 its initial value is 0 (inconsistent).
  - Round 2: All truthful.
- $\alpha_3 \sim^1 \alpha_1$, so p1 behaves the same in both, decides 1 in α_3.
- $\alpha_3 \sim^3 \alpha_2$, so p3 behaves the same in both, decides 0 in α_3.
- Contradicts agreement!

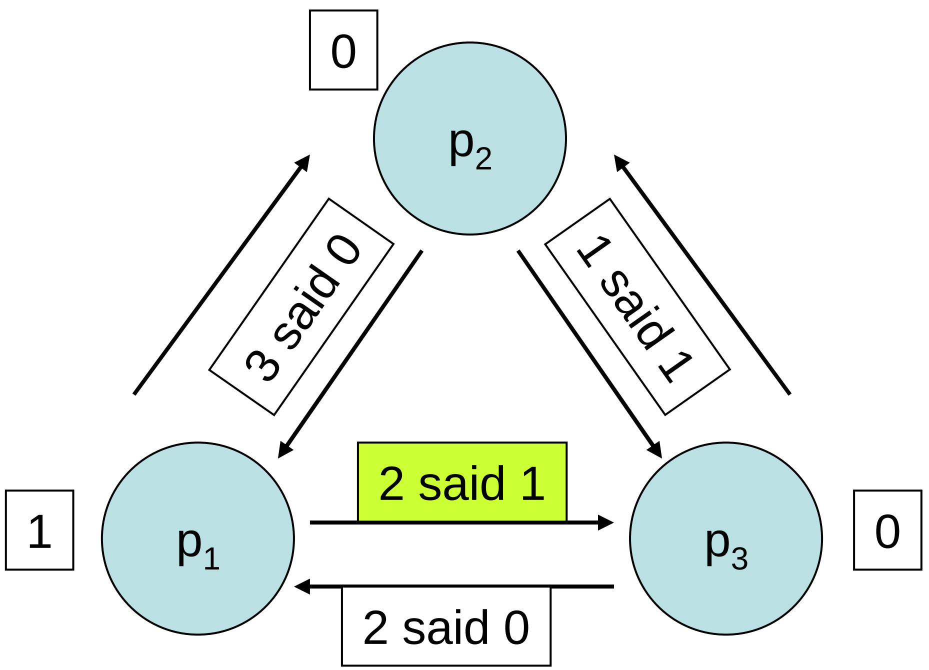
Bad example

- α_1: p1 and p2 nonfaulty, initial value 1, p3 faulty, initial value 0
  - Round 1: All truthful
  - Round 2: p3 lies, telling p1 that "p2 said 0"; all other communications are truthful.
  - Validity requires that p1 and p2 decide 1.



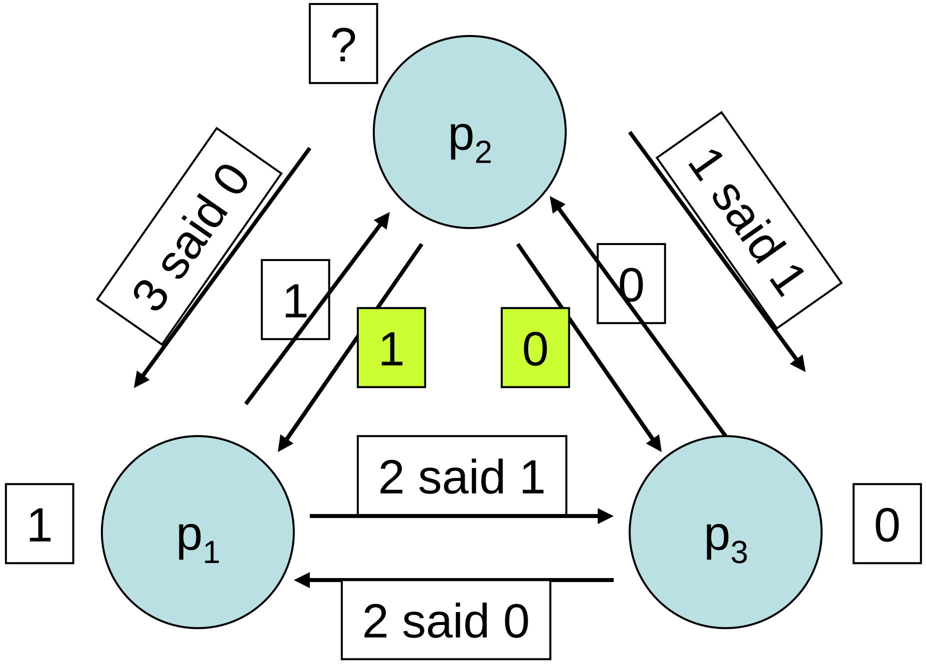
Bad example

- α_2: p2 and p3 nonfaulty, initial value 0, p1 faulty, initial value 1
  - Round 1: All truthful
  - Round 2: p1 lies, telling p3 that "p2 said 1"; all other communications are truthful.
  - Validity requires that p2 and p3 decide 0.



Bad example

- α_3: p1 nonfaulty, initial value 1, p3 nonfaulty, initial value 0, p2 faulty, initial value doesn't matter.
  - Round 1: p2 tells p1 its initial value is 1, tells p3 its initial value is 0 (inconsistent).
  - Round 2: All truthful.



Notes on the example

- The correct processes can tell something is wrong, but that doesn't help:
  - E.g., in α_1, p1 sees that p2 sends 1, but p3 said that p2 said 0.
  - So p1 knows that either p2 or p3 is faulty, but doesn't know which.
  - By termination, p1 has to decide something, but neither value works right in all cases.

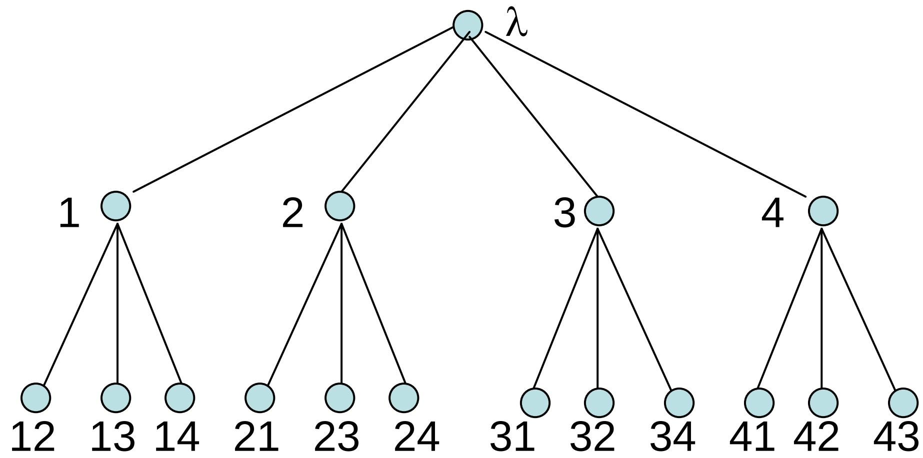
- Impossibility of solving Byzantine agreement with 3 processes, 1 failure:
  - This is not a proof--- maybe there's a non-EIG algorithm, or one that takes more rounds,…
  - Come back to this later.

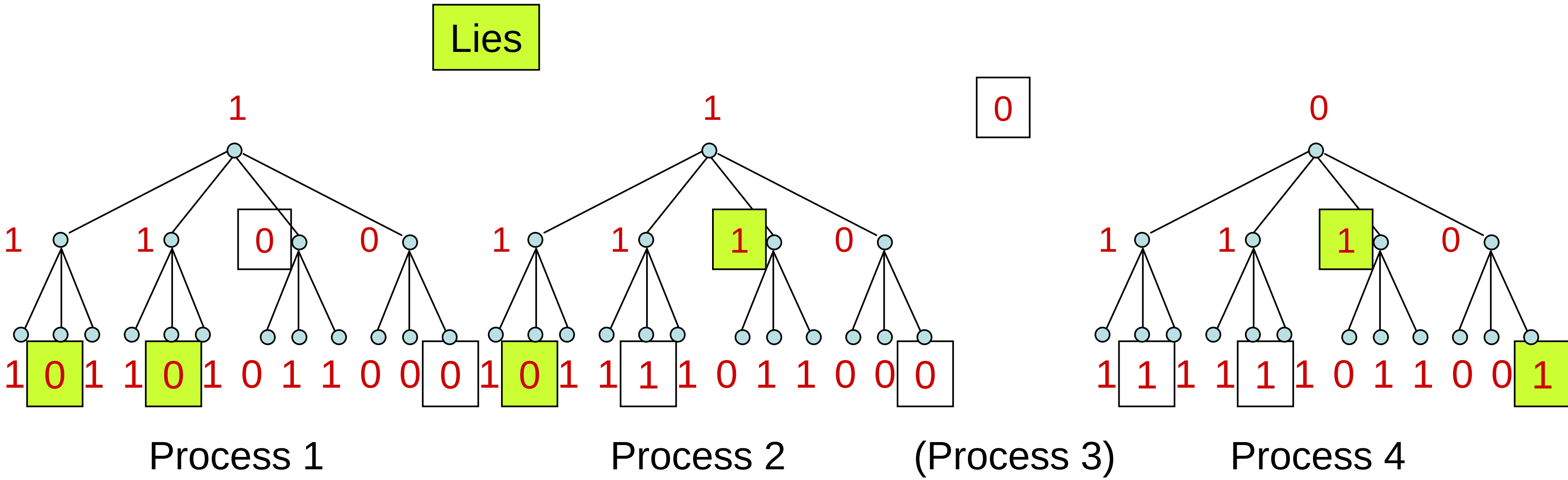
EIG algorithm for Byzantine agreement

- Assume $n > 3f$.
- Same EIG tree as before.
- Relay messages for f+1 rounds, as before.
- Decorate the tree with values from V, replacing any garbage messages with default value v_0.
- New decision rule:
  - Call the decorations val(x), where x is a node label.
  - Redecorate the tree, defining newval(x).
    - Proceed bottom-up.
    - Leaf: newval(x) = val(x)
    - Non-leaf: newval(x) =
      - newval of strict majority of children in the tree, if majority exists,
      - v_0 otherwise.
  - Final decision: newval(λ) (newval at root)

Example: n = 4, f = 1

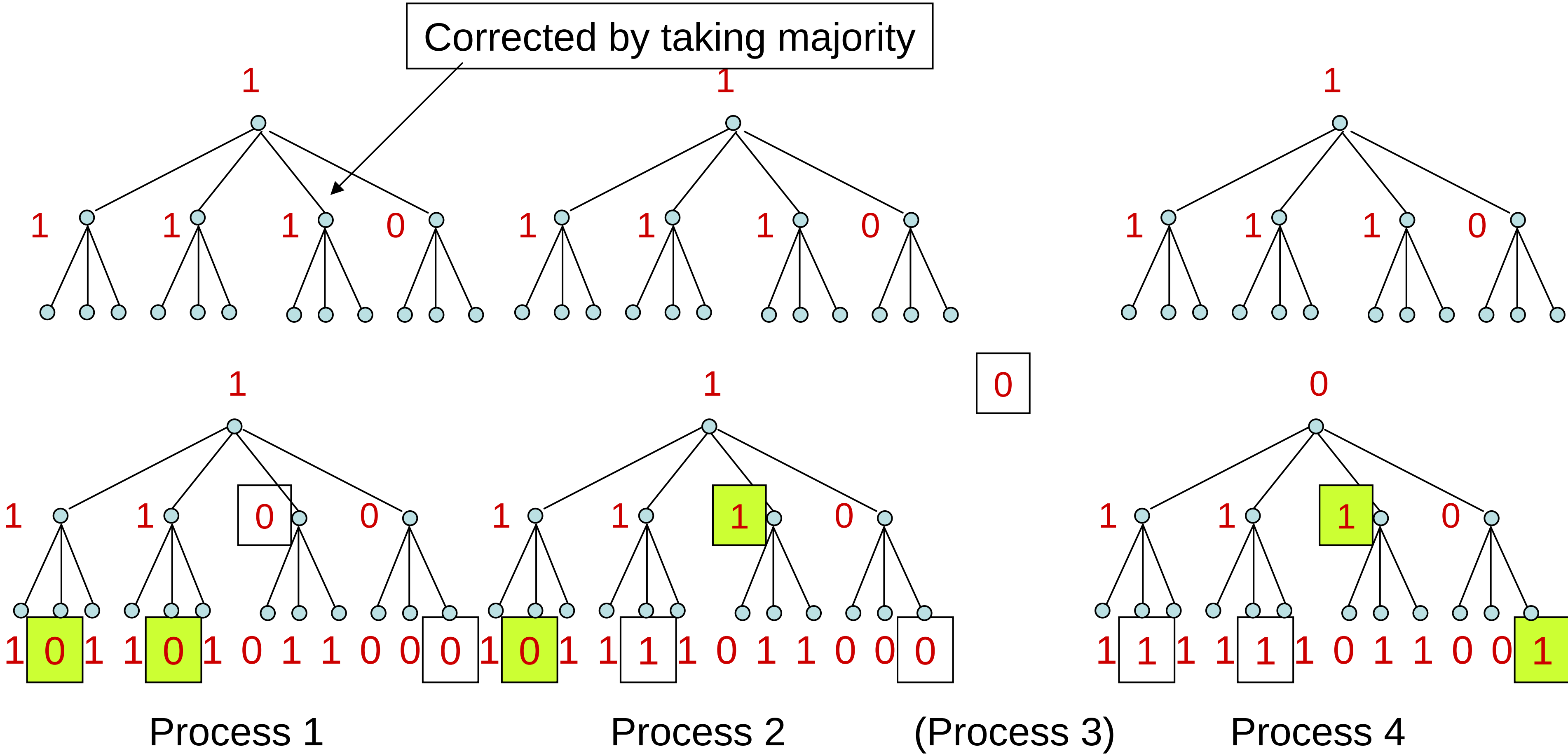
- $T_{4,1}$:
- Consider a possible execution in which p3 is faulty.
- Initial values 1 1 0 0
- Round 1
- Round 2





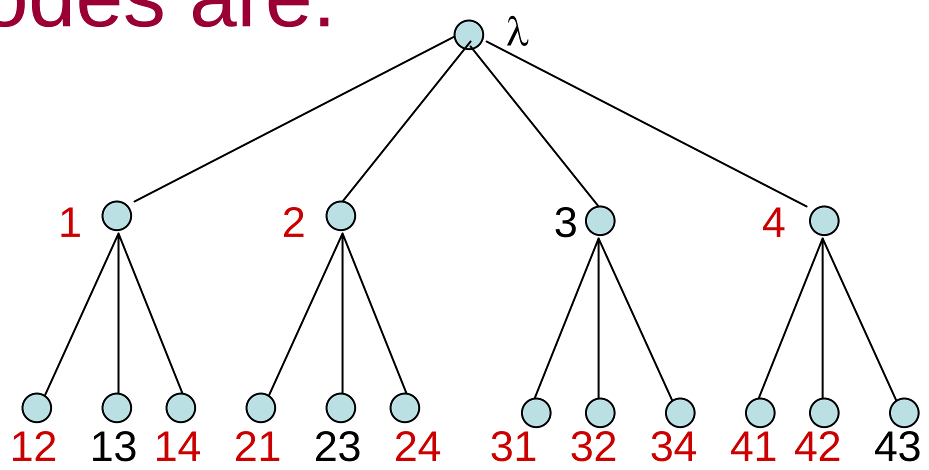
Example: n = 4, f = 1

- Now calculate newvals, bottom-up, choosing majority values, $v_0 = 0$ if no majority.



Correctness proof

- Lemma 1: If i, j, k are nonfaulty, then $val(x)_i = val(x)_j$ for every node label x ending with k.
- In example, such nodes are:



- Proof: k sends same message to i and j and they decorate accordingly.

Proof, cont'd

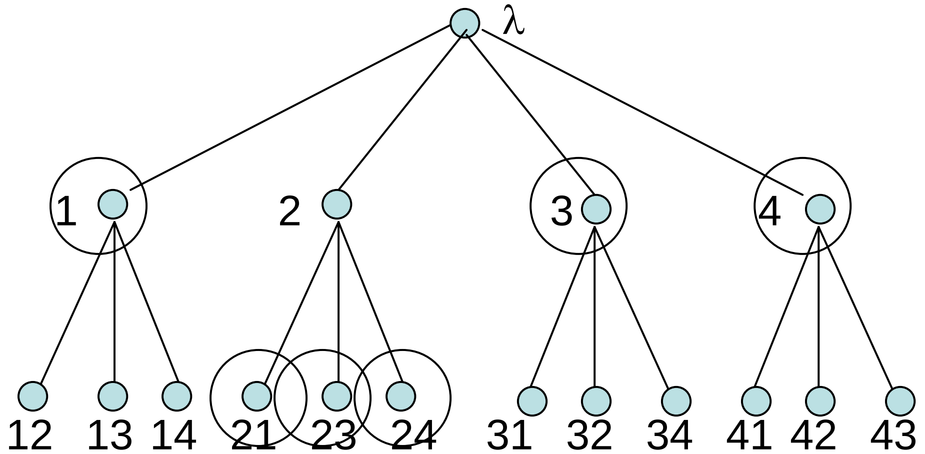
- Lemma 2: If x ends with nonfaulty process index then $\exists v \in V$ such that $val(x)_i = newval(x)_i = v$ for every nonfaulty i.
- Proof: Induction on lengths of labels, bottom up.
  - Basis: Leaf.
    - Lemma 1 implies that all nonfaulty processes have same val(x).
    - newval = val for each leaf.
  - Inductive step: $|x| = r \leq f$ ($|x| = f+1$ at leaves)
    - Lemma 1 implies that all nonfaulty processes have same val(x), say v.
    - We need newval(x) = v everywhere also.
    - Every nonfaulty process j broadcasts same v for x at round r+1, so $val(xj)_i = v$ for every nonfaulty j and i.
    - By inductive hypothesis, also $newval(xj)_i = v$ for every nonfaulty j and i.
    - A majority of labels of x's children end with nonfaulty process indices:
      - Number of children of node x is $\geq n - f > 3f - f = 2f$.
      - At most f are faulty.
    - So, majority rule applied by i leads to $newval(x)_i = v$, for all nonfaulty i.

Main correctness conditions

- Validity:
  - If all nonfaulty processes begin with v, then all nonfaulty processes broadcast v at round 1, so $val(j)_i = v$ for all nonfaulty i, j.
  - By Lemma 2, also $newval(j)_i = v$ for all nonfaulty i,j.
  - Majority rule implies $newval(\lambda)_i = v$ for all nonfaulty i.
  - So all nonfaulty i decide v.
- Termination:
  - Obvious.
- Agreement:
  - Requires a bit more work:

Agreement

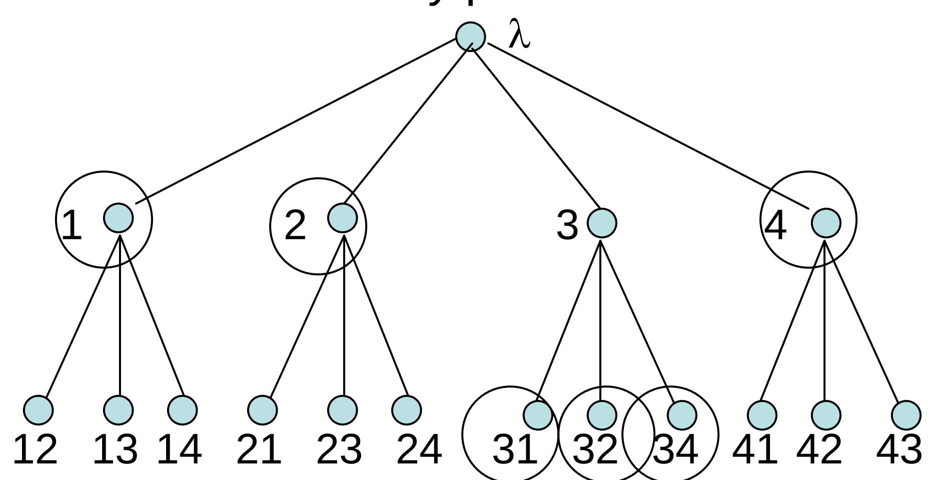
- Path covering: Subset of nodes containing at least one node on each path from root to leaf:



- Common node: One for which all nonfaulty processes have the same newval.
  - If a node's label ends in nonfaulty process index, Lemma 2 implies it's common.
  - Others might be common too.

Agreement

- Lemma 3: There exists a path covering all of whose nodes are common.
- Proof:
  - Let C = nodes with labels of the form xi, i nonfaulty.
  - By Lemma 2, all of these are common.
  - Claim these form a path covering:
    - There are at most f faulty processes.
    - Each path contains f+1 labels ending with f+1 distinct indices.
    - So at least one of these labels ends with a nonfaulty process index.



Agreement

- Lemma 4: If there's a common path covering of the subtree rooted at any node x, then x is common
- Proof:
  - By induction, from the leaves up.
  - "Common-ness" propagates upward.

- Lemma 5: The root is common.
- Proof: By Lemmas 3 and 4.

- Thus, all nonfaulty processes get the same newval(λ).
- Yields Agreement.

Complexity bounds

- As for EIG for stopping agreement:
  - Time: f+1
  - Communication: $O(n^{f+1})$
- Number of processes: $n > 3f$

Next time…

- Lower bounds for Byzantine agreement:
  - Number of processors
  - Bounds for connectivity, weak Byzantine agreement.
  - Number of rounds
- Reading:
  - Sections 6.4-6.7
  - [Aguilera, Toueg]
  - (Optional) [Keidar-Rajsbaum]

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