Session 13 *(In preparation for Class 13, students are asked to view Lecture 13.)*

Topics for Class 13

Locked linkages: Why expansiveness, energy algorithm correctness, pointed pseudotriangulations (combinatorics, rigidity, universality, expansiveness, extremeness), linear equilateral trees can't lock, unfolding 4D chains.

Detailed Description of Class 13

This class covers several interesting results about pointed pseudotriangulations:

- Their original use for polygon ray shooting data structures.
- Their (fixed) number of edges and faces (in contrast to triangulations).
- Their minimal generic rigidity
- Every planar minimally generically rigid graph can be drawn as a pointed pseudotriangulation (a kind of universality)
- Why they work as an algorithm for the Carpenter's Rule Theorem: they have expansive motions after removing a convex-hull edge. (In particular, we'll review some lemmas from CDR.)
- In fact, these expansive motions are the "extreme rays" (edges) of the cone of all expansive motions.

In addition, we cover the following questions and results:

- Why do we use expansiveness? Both convenience and mathematical power.
- Why can't the energy algorithm get stuck in a local minimum?
- New result: linear equilateral trees can't lock
- Old result: 4D (and higher-dimensional) open chains can't lock

Topics for Lecture 13

Locked linkages: Algorithms for unfolding 2D chains, pseudotriangulation, energy; rigid folding of single-vertex origami; locked trees, infinitesimally locked linkages, Rules 1 and 2; locked 3D chains, knitting needles.

Detailed Description of Lecture 13

This lecture is about locked linkages. Continuing on from the Carpenter's Rule Theorem from last lecture, which says that 2D chains can't lock, we'll see three different algorithms for folding 2D chains. Each algorithm has varying levels of expansiveness, symmetry, and efficiency, with applications to 2D robot-arm motion planning. We'll also see an application of a spherical version of the Carpenter's Rule Problem to rigid folding of single-vertex origami. Then we'll tour the world of locked 2D trees, which has had significant progress recently. To this end, I'll describe the extensive technology for proving 2D linkages to be locked. Finally we'll briefly look at locked 3D chains, which relates to protein folding.

6.849 Geometric Folding Algorithms: Linkages, Origami, Polyhedra
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