Beyond Verification

Software Synthesis

What do we mean by synthesis

We want to get code from high-level specs

- Python and VB are pretty high level, why is that not synthesis?

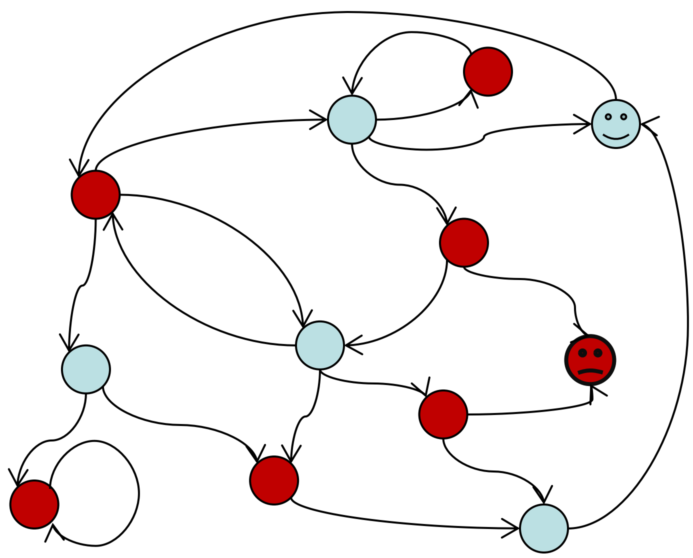
Support compositional and incremental specs

- Python and VB don't have this property
  - If I don't like the way the python compiler/runtime is implementing my program, I am out of luck.
- Logical specifications do
  - I can always add additional properties that my system can satisfy
- Specs are not only functional
  - Structural specifications play a big role in synthesis
  - How is my algorithm going to look like.

The fundamental challenge

The fundamental challenge of synthesis is dealing with an uncooperative environment

- For reactive systems, people model this as a game
  - For every move of the adversary (ever action of the environment), the synthesized program must make a counter-move that keeps the system working correctly.
  - The game can be modeled with an automata



The fundamental challenge

The fundamental challenge of synthesis is dealing with an uncooperative environment

- If we are synthesizing functions, the environment provides the inputs
  - i.e. whatever we synthesize must work correctly for all inputs

- This is modeled with a doubly quantified constraint
  - E.g. if the spec is given as pre and post conditions, we have

$$\exists P \, \forall \sigma \quad (\sigma \vDash \{pre\}) \Rightarrow (\sigma \vDash WP(P, \{post\}))$$

- What does it mean to quantify over the space of programs?

Quantifying over programs

Synthesis in the functional setting can be seen as curve fitting

- i.e. we want to find a curve that satisfies some properties

It's very hard to do curve fitting when you have to consider arbitrary curves

- Instead, people use *parameterized* families of curves
- This means you quantify over parameters instead of over functions

This is the first fundamental idea in software synthesis

- People call these Sketches, scaffolds, templates, …
- They are all the same thing

The Sketch Language

Define parameterized programs explicitly

- Think of the parameterized programs as "programs with holes"

Example: Hello World of Sketching

spec:

```
int foo (int x)
{
    return x + x;
}
```

sketch:

```
int bar (int x) implements foo
{
    return x * ??;
}
```

Integer Hole

Integer Holes → Sets of Expressions

Expressions with **??** == sets of expressions

- linear expressions `x*??  + y*??`
- polynomials `x*x*??  + x*??  + ??`
- sets of variables `?? ? x : y`

Integer Holes → Sets of Expressions

Example: Least Significant Zero Bit

- 0010 0101 → 0000 0010

```
int W = 32;

bit[W] isolate0 (bit[W] x) {      // W: word size
       bit[W] ret = 0;
       for (int i = 0; i < W; i++)
              if (!x[i]) { ret[i] = 1; return ret;  }
}
```

Trick:

- Adding 1 to a string of ones turns the next zero to a 1
- i.e. 000111 + 1 = 001000

`!(x + ??) & (x + ??)` → `!(x + 1) & (x + 0)`, `!(x + 1) & (x + 0xFFFF)`, `!(x + 0) & (x + 1)`, `!(x + 0xFFFF) & (x + 1)`

Integer Holes → Sets of Expressions

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int W = 32;

bit[W] isolate0 (bit[W] x) {      // W: word size
        bit[W] ret = 0;
        for (int i = 0; i < W; i++)
                if (!x[i]) { ret[i] = 1; return ret;  }
}

bit[W] isolateSk (bit[W] x) implements isolate0 {

        return !(x + ??) & (x + ??) ;
}
```

Integer Holes → Sets of Expressions

Expressions with **??** == sets of expressions

- linear expressions `x*?? + y*??`
- polynomials `x*x*?? + x*?? + ??`
- sets of variables `?? ? x : y`

Semantically powerful but syntactically clunky

- Regular Expressions are a more convenient way of defining sets

Regular Expression Generators

{| RegExp |}

RegExp supports choice '|' and optional '?'

- can be used arbitrarily within an expression
  - to select operands `{| (x | y | z) + 1 |}`
  - to select operators `{| x (+ | -) y |}`
  - to select fields `{| n(.prev | .next)? |}`
  - to select arguments `{| foo( x | y, z) |}`

Set must respect the type system

- all expressions in the set must type-check
- all must be of the same type

Least Significant One revisited

How did I know the solution would take the form

`!(x + ??) & (x + ??) .`

What if all you know is that the solution involves `x`, `+`, `&` and `!`.

```
bit[W] tmp=0;
{| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
{| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
return tmp;
```

This is now a set of statements
(and a really big one too)

Sets of statements

Statements with holes = sets of statements

Higher level constructs for Statements too

- repeat

```
bit[W] tmp=0;
repeat(3){
    {| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
}
return tmp;
```

repeat

Avoid copying and pasting

- `repeat(n){ s}` ➔ `s;s;…s;` (n copies)
- each of the n copies may resolve to a distinct stmt
- n can be a hole too.

```
bit[W] tmp=0;
repeat(??){
    {| x | tmp |} = {| (!)?((x | tmp) (& | +) (x | tmp | ??)) |};
}
return tmp;
```

Keep in mind:

- the synthesizer won't try to minimize n

Solving for a parameterized program

At a high level, two fundamental approaches:

- Search and Test

- Derive in one shot
  - Usually by means of abstraction.

The CEGIS approach

Synthesis reduces to constraint satisfaction

$$\exists\, \phi. \quad \forall\, x. \quad Q(x, \phi)$$

Constraints are too hard for standard techniques

- Universal quantification over inputs
- Too many inputs
- Too many constraints
- Too many holes

Insight

Sketches are not arbitrary constraint systems

- They express the high level structure of a program

A small number of inputs can be enough

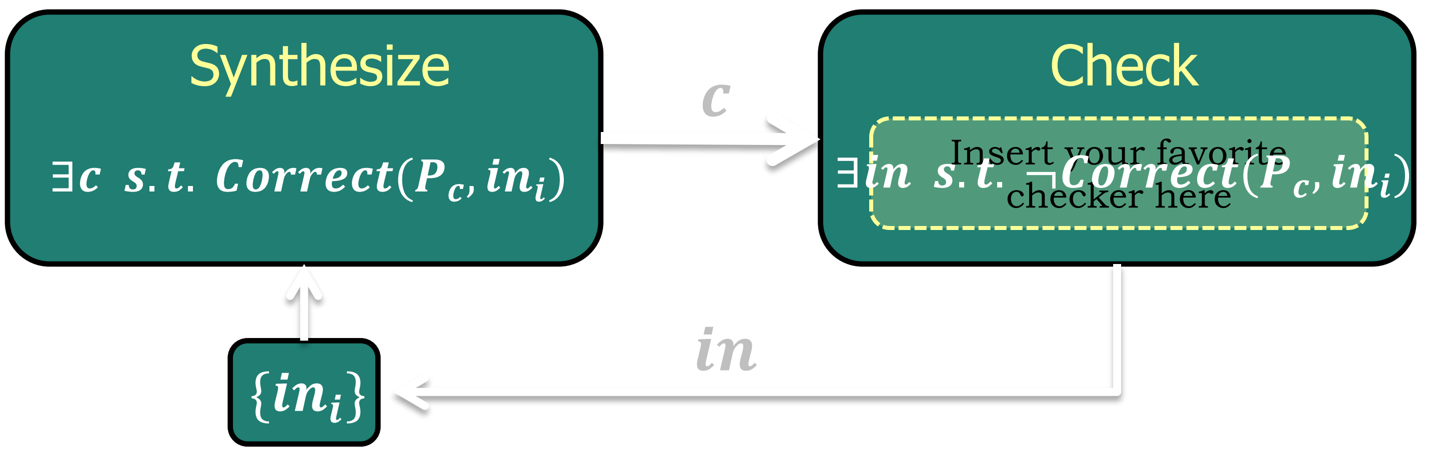
- focus on corner cases

$$\exists \; \phi . \; \forall x \text{ in } E. \; Q(x, \phi)$$

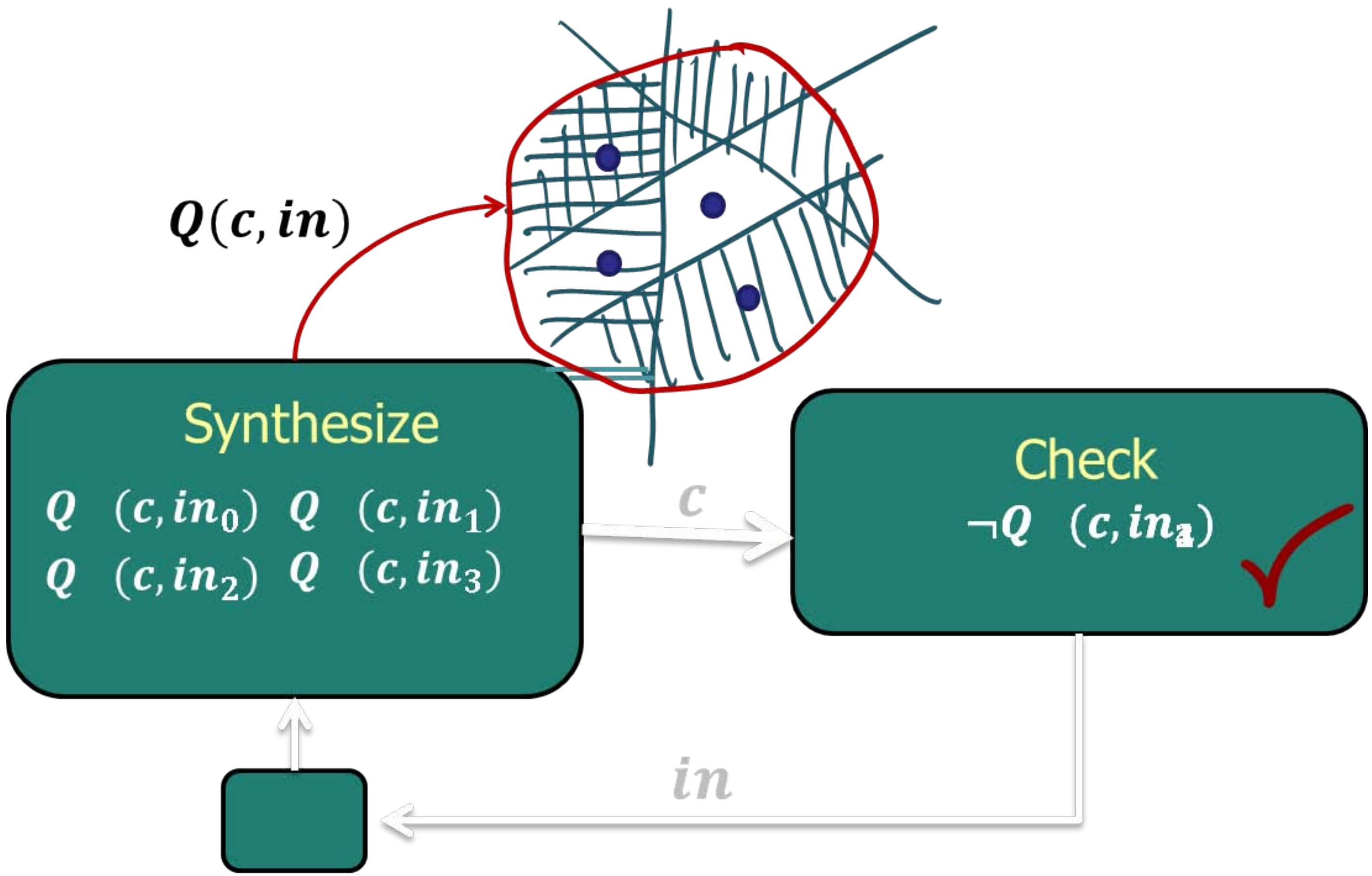
where $E = \{x_1, x_2, \ldots, x_k\}$

This is an inductive synthesis problem !

CEGIS Synthesis algorithm



CEGIS



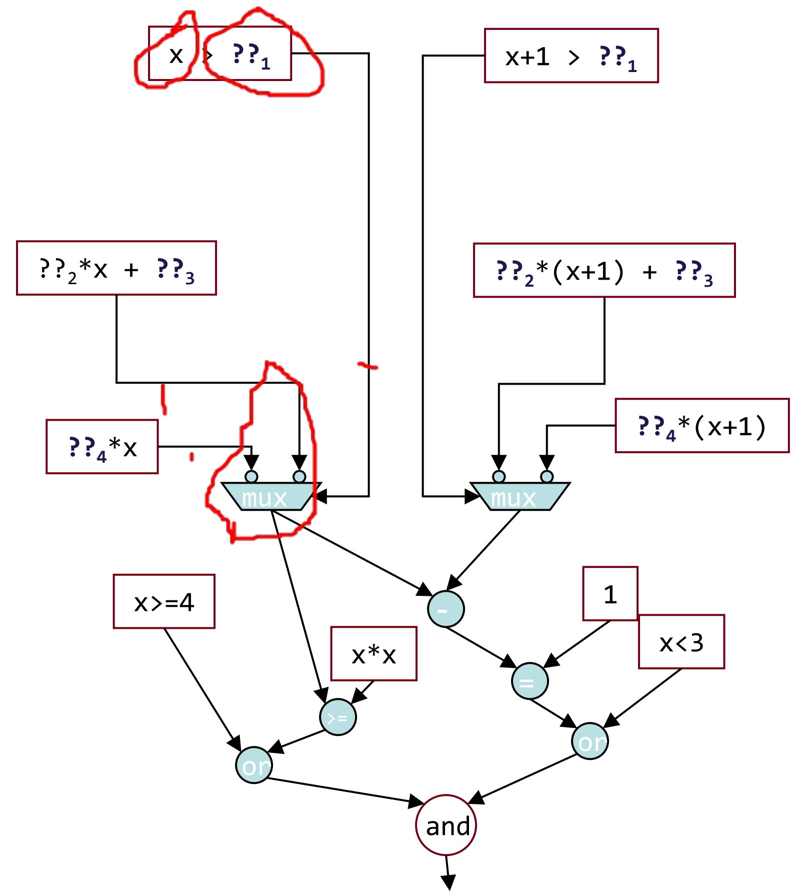
A sketch as a constraint system

```
int lin(int x){
   if(x > ??₁)
      return ??₂*x + ??₃;
   else
      return ??₄*x;
}

void main(int x){
   int t1 = lin(x);
   int t2 = lin(x+1);

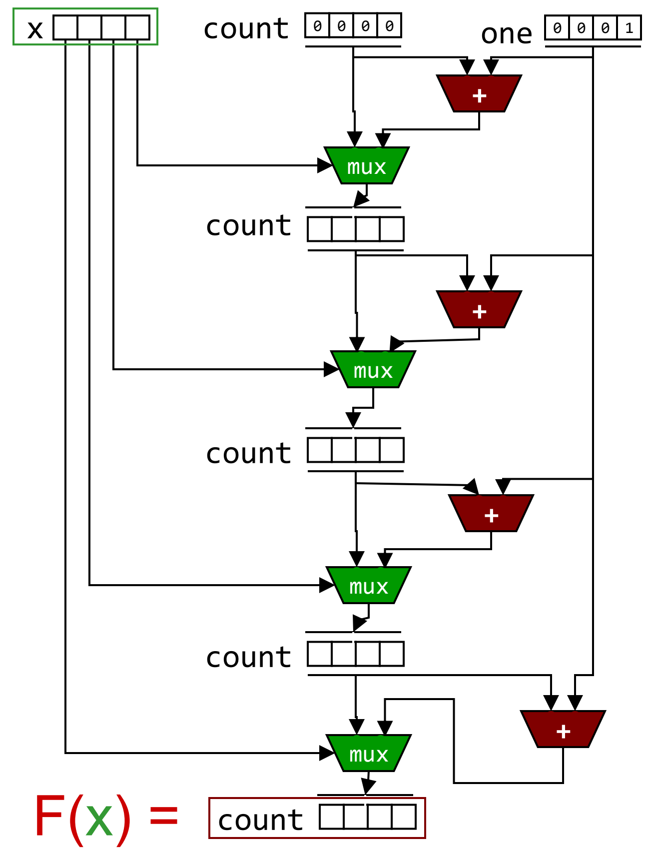
   if(x<4) assert t1 >=  x*x;

   if(x>=3) assert t2-t1 == 1;
}
```



Ex : Population count. 0010 0110 → 3

```
int pop (bit[W] x)
{
  int count = 0;
  for (int i = 0; i < W; i++) {
    if (x[i]) count++;
  }
  return count;
}
```



```
int popSketched (bit[W] x)
    implements pop {
    repeat(??) {
            x = (x & ??)
            + ((x >> ??) & ??);
    }
    return x;
}
```