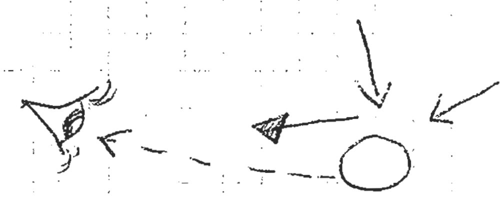
6.866

I Radiance $E = \frac{dP}{dA}$ $(W . m^{-2})$

Scene Radiance $L = \frac{d^2 P}{dA\, d\omega}$ $(W . m^{-2} sr^{-1})$

Lens : $E = L \frac{\pi}{4} \left(\frac{d}{f}\right)^2 \cos^4 \alpha$

L ? ① Illumination
(a) irradiance
(b) directionality / geometry



② Surface material
(a) albedo 0–1
(b) surface orientation

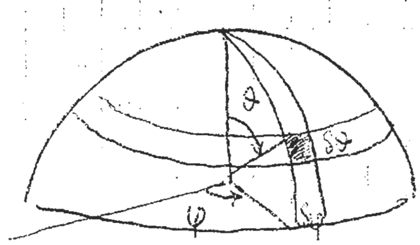
③ Viewer
(a) sensitivity
(b) direction / geometry

BRDF $f(\theta_i, \varphi_i ; \theta_e, \varphi_e) = \frac{\delta L(\theta_e, \varphi_e)}{\delta E(\theta_i, \varphi_i)}$

- Extended light sources

$\delta s = \delta\theta\, \delta\varphi \sin\theta$

$E(\theta_i, \varphi_i)$ radiance per unit solid angle



Total light, $E_0 = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} E(\theta_i, \varphi_i) \sin\theta_i \cos\theta_i \, d\theta_i \, d\varphi_i$

Light received, $L = L(\theta_e, \varphi_e)$

$$L(\theta_e, \varphi_e) = \int_{-\pi}^{\pi} \int_{0}^{\pi/2} f(\theta_i, \varphi_i, \theta_e, \varphi_e) \, E(\theta_i, \varphi_i) \sin\theta_i \cos\theta_i \, d\theta_i \, d\varphi_i$$

(⚠ : lense is considered as a pinhole
→ does not work for a microscope !)

To get albedo : integrate BRDF over (θ_e, φ_e)

- Ideal Lambertian Surfaces :
  1. reflects all incident light
  2. appear equally bright viewed from all directions

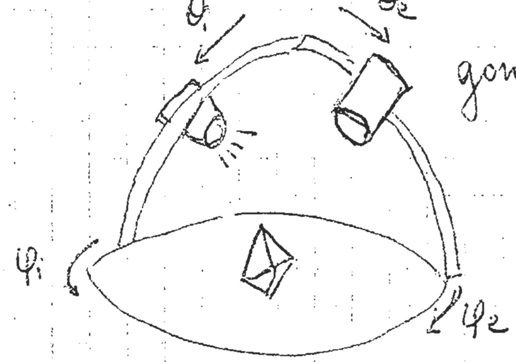
1 $\Rightarrow$ albedo = 1 2 $\Rightarrow$ f constant

$$\int_{-\pi}^{\pi} \int_{0}^{\frac{\pi}{2}} f \cdot E \cos\theta_e \sin\theta_e \cos\theta_i \sin\theta_i \, d\theta_e \, d\varphi_e = E \cos\theta_i$$

$$2\pi f \int_0^{\pi/2} \sin\theta_e \cos\theta_e \, d\theta_e = 1 \quad \Rightarrow \quad \boxed{f = \frac{1}{\pi}}$$

BRDF ?

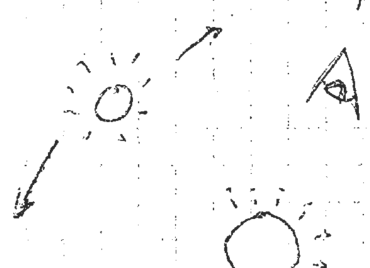
① 1 way is to measure



goniometer

4 DOF

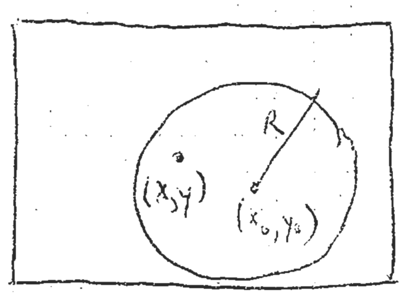
② Another way: use a calibration object



(in a single image, we have info for all directions)

The image is a sphere

2 DOF



$$\vec{M} = \begin{vmatrix} x - x_0 \\ y - y_0 \\ \sqrt{R^2 - (x - x_0)^2 - (y - y_0)^2} \end{vmatrix}$$

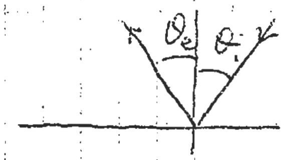
③ Third way: phenomenological models

ex: Lambertian $\cos\theta_i$ ↳ defined by its properties

(ii) Hapke : $\sqrt{\dfrac{\cos\theta_i}{\cos\theta_e}}$

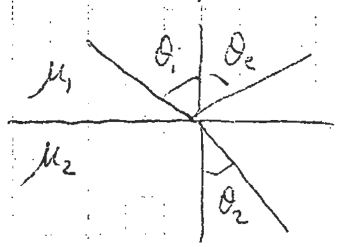
(iii) Fesenkov :

(iv) Physical models

① 

specular

$\theta_i = \theta_e$, etc...

② 

dielectric (glass, water)

Snell's law Fresnel reflection

③ microstructure

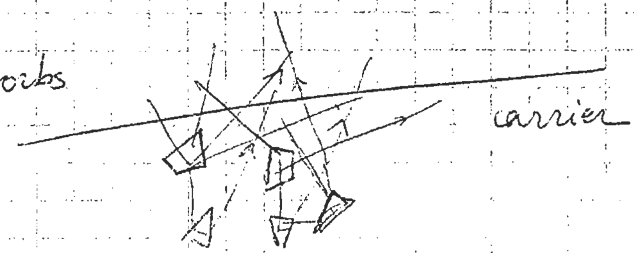
regular pattern

random pattern

- case 1 : closed form
- case 2 : Monte Carlo methods

ex: white paint

- pigment selectively absorbs
- diffusion & refraction



- Silicon dioxid (glass) SiO_2
- lots of particles per unit volume $\Longrightarrow$ "hiding power"
- $\rightarrow$ particles small (but $> \lambda$)

$\boxed{\text{RA: Chapter 10}}$

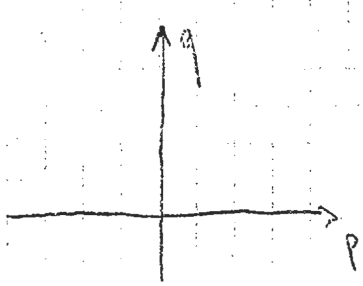
Surface orientation $\left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)$ gradient $\delta z = p\,\delta x + q\,\delta y + \ldots$

unit normal: $\underline{\hat{n}} = \frac{1}{\sqrt{1+p^2+q^2}} \begin{vmatrix} -p \\ -q \\ 1 \end{vmatrix}$

Reflectance Map

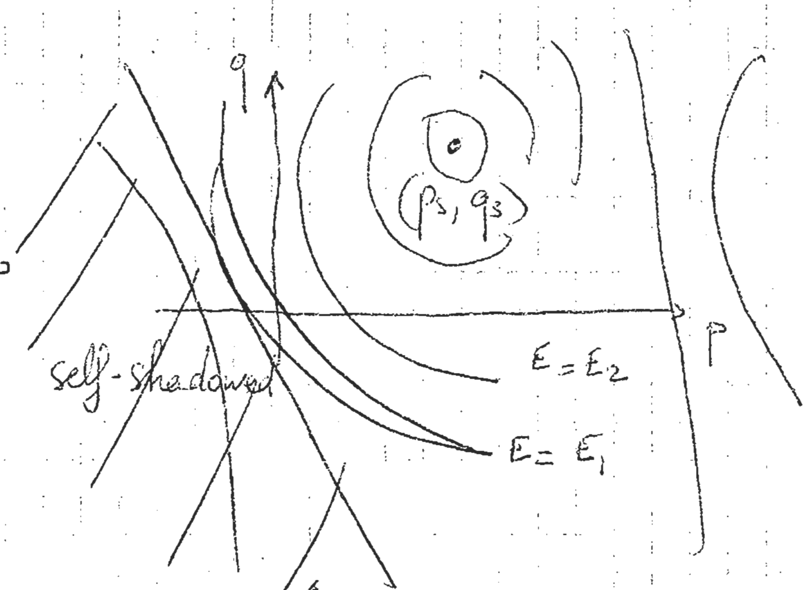
radiance as a function of orientation $R(p,q)$

Lambertian: $\cos\theta_i = \underset{\sim}{\hat{n}} \cdot \underset{\sim}{\hat{s}}$

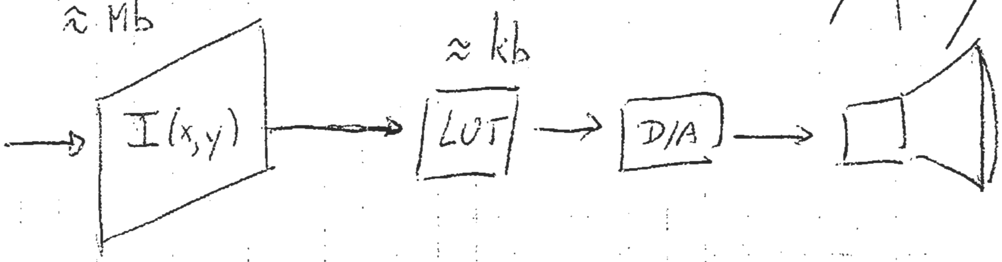


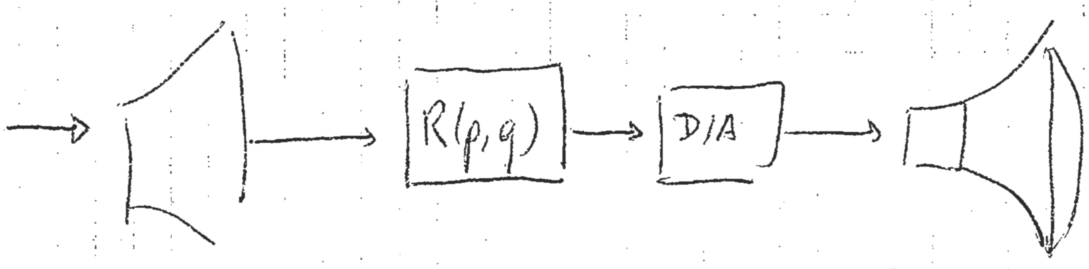
$\underline{\hat{n}} = \frac{(-p, -q, 1)^T}{\sqrt{1+p^2+q^2}}$ $\qquad$ $\underline{\hat{s}} = \frac{(-p_s, -q_s, 1)^T}{\sqrt{1+p_s^2+q_s^2}}$

$$R(p,q) = \frac{1 + p_s p + q_s q}{\sqrt{1+p^2+q^2}\;\underbrace{\sqrt{1+p_s^2+q_s^2}}_{r_s}}$$



maximum: $(p,q) = (p_s, q_s)$





p	q
1011	0011